Lesson 1.1 – Multiplication of Numbers in Exponential Form

Recall that exponents group common bases that are connected by multiplication. Fill in the missing exponents below.

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^{(5)}$ $\qquad$ $5 \cdot 5 \cdot a \cdot a \cdot a \cdot b = 5^{(2)} a^{(3)} b^{(1)}$

$\underbrace{11 \times \cdots \times 11}_{\text{32 times}} = 11^{(32)}$ $\qquad$ $\underbrace{(-3) \times \cdots \times (-3)}_{\text{17 times}} = (-3)^{(17)}$

Also, when an expression is in exponential notation, understanding how to use an exponent allows us to expand the expression. The bases below have large exponents. For the expressions, state the amount of times you would expect to see the base if we were to expand it.

$4^{19} = \underbrace{4 \times \cdots \times 4}_{\underline{19}\text{ times}}$ $\qquad$ $15^{48} = \underbrace{15 \times \cdots \times 15}_{\underline{48}\text{ times}}$

$5.7^{23} = \underbrace{5.7 \times \cdots \times 5.7}_{\underline{23}\text{ times}}$ $\qquad$ $\left(\frac{3}{4}\right)^{101} = \underbrace{\left(\frac{3}{4}\right) \times \cdots \times \left(\frac{3}{4}\right)}_{\underline{101}\text{ times}}$

In general, for any number y and any positive integer n, $$y^n = \underbrace{y \times \cdots \times y}_{n\text{ times}}$$	The number y^n is called **y raised to the n-th power,** n is the **exponent** of y in y^n and y is the **base** of y^n.

Multiplication of Numbers in Exponential Form

$$12^7 \times 12^2$$

An example of numbers in exponential form being multiplied.

What happens when we multiply like powers connected by multiplication in an expression?

Having a deep understanding on how to expand or condense an expression using exponents is critical to helping us answer that question. Complete the exercises on the next page.

Class Notes – Expand the expression, then condense it.

LP#1 $4^3 \cdot 4^5$ 4^8	$8^2 \cdot 8^4$ 8^6
LP#2 $x^2 \cdot x^3 \cdot x^5$ x^{10}	$y^3 \cdot y^6 \cdot y$ y^{10}
LP#3 $3^3 \cdot 7 \cdot 3^4 \cdot 7^2$ $3^7 \cdot 7^3$	$6^2 \cdot 9^4 \cdot 6^3 \cdot 9^2$ $6^5 \cdot 9^6$

Use what you observe above to complete the following. Combine the like powers below into one power.

$x^a \cdot x^b = x^{a+b}$	$x^a \cdot y^c \cdot x^b \cdot y^d = x^{a+b} \cdot y^{c+d}$

When we multiply like bases the exponent for the new expression is the (sum/difference/product/quotient) sum of the original exponents.

Class Notes – Simplify the following expressions. Show your work by using one of the two methods below.

Examples of how to show your work.

Expanding and Condensing	Using the algorithm
$3^4 \times 3^2 = 3 \cdot 3 \cdot 3 \cdot 3 \times 3 \cdot 3$	$3^4 \times 3^2 = 3^{4+2}$
$= 3^6$	$= 3^6$

LP#4 $7^6 \cdot 7^8$ 7^{14}	$3^8 \cdot 3^{10}$ 3^{18}	$x^4 \cdot x^7$ x^{11}	$m^9 \cdot m^3$ m^{12}
LP#5 $a^2 \cdot b^2 \cdot a^4$ $a^6 \cdot b^2$	$6^5 \cdot y \cdot y^9 \cdot 6^4$ $6^9 \cdot y^{10}$	$11^2 \cdot 11 \cdot w^2 \cdot 11^5$ $11^8 \cdot w^2$	$5^3 \cdot 5^6 \cdot p^3 \cdot p$ $5^9 p^4$

Review – Simplify the following expressions. Show work.

R#1 $9^8 \cdot 9^{10}$ 9^{18}	$5^4 \cdot 5^6$ 5^{10}	$b^7 \cdot b^9$ b^{16}	$a \cdot a^4 \cdot a^4$ a^9
R#2 $7 \cdot 7^5$ 7^6	$p^5 \cdot p^2$ p^7	$k^2 \cdot k^2 \cdot k^4$ k^8	$x^3 \cdot y \cdot y^2 \cdot x^4$ $x^7 y^3$
R#3 $12^{11} \cdot 12^9$ 12^{20}	$a^8 \cdot b^4 \cdot a^5$ $a^{13} b^4$	$k^3 \cdot m^5 \cdot m^7$ $k^3 m^{12}$	$3^3 \cdot y \cdot y^5 \cdot 3^4$ $3^7 y^6$

Lesson 1.2 – Division of Numbers in Exponential Form

Recall that when we combined bases that are being multiplied we add the exponents.
Let's take a closer look at this rule.

In general, if y is any number and m, n are positive integers, then

$$y^m \cdot y^n = y^{m+n}$$

because

$$y^m \times y^n = \underbrace{(y \times \cdots \times y)}_{m \text{ times}} \times \underbrace{(y \times \cdots \times y)}_{n \text{ times}} = \underbrace{(y \times \cdots \times y)}_{m+n \text{ times}} = y^{m+n}$$

Division of Numbers in Exponential Form

$$4^7 \div 4^2 \qquad \frac{9^{15}}{9^{11}}$$

Two examples of numbers in exponential form being divided.

We determined what occurs when we multiply numbers in exponential form, what occurs when we divide numbers in exponential form?

Let's use some logic. When multiplying like bases we add the exponents. Division is the opposite operation of multiplication, so what is the opposite operation of addition? subtraction

We can hypothesize that we subtract exponents when we divide numbers in exponential form. We will put that hypothesis to a test. Complete the notes on the next page.

Class Notes – Similar to the last lesson, we will expand the expression, then condense it.

LP#1 $\frac{8^7}{8^3}$ 8^4	$\frac{12^6}{12^4}$ 12^2
LP#2 $\frac{x^5}{x^2}$ x^3	$\frac{y^{10}}{y^5}$ y^5
LP#3 $\frac{4^3 \cdot m^6}{4 \cdot m^4}$ $4^2 \cdot m^2$	$\frac{7^5 \cdot p^4}{7^3 \cdot p}$ $7^2 \cdot p^3$

Use what you observe above to complete the following. Combine the like powers below into one power.

$\frac{x^a}{x^b} = x^{a-b}$	$\frac{x^a \cdot y^c}{x^b \cdot y^d} = x^{a-b} y^{c-d}$

When we divide like bases the exponent for the new expression is the (sum/difference/product/quotient) difference of the original exponents.

Class Notes – Simplify the following expressions. Show your work by using one of the two methods below.

Examples of how to show your work.

Expanding and Condensing

$$3^5 \div 3^2 = \frac{3^5}{3^2} = \frac{3\cdot3\cdot3\cdot3\cdot3}{3\cdot3}$$

$$= \frac{3\cdot3\cdot3\cancel{\cdot3\cdot3}}{\cancel{3\cdot3}}$$

$$= 3\cdot3\cdot3$$

$$= 3^3$$

Using the algorithm

$$3^5 \div 3^2 = \frac{3^5}{3^2}$$

$$= 3^{5-2}$$

$$= 3^3$$

LP#4 $\frac{13^{11}}{13^7}$ 13^4	$\frac{20^{19}}{20^{10}}$ 20^9	$\frac{190^{32}}{190^{14}}$ 190^{18}	$\frac{d^{15}}{d^8}$ d^7
LP#5 $\frac{g^3h^{24}}{gh^{16}}$ g^2h^8	$\frac{j^4k^{55}}{j^2k^{22}}$ j^2k^{33}	$\frac{6^7\cdot x^{16}}{6^2\cdot x^{11}}$ 6^5x^5	$\frac{14^{10}\cdot y^{16}}{14^4\cdot y^4}$ 14^6y^8

Review – Simplify the following expressions. Show work.

R#1 $\frac{10^8}{10^4}$ 10^4	$\frac{m^5}{m^2}$ m^3	$\frac{k^{14}}{k^{10}}$ k^4	$\frac{a^7 \cdot b^{16}}{a^2 \cdot b^{11}}$ $a^5 b^5$
R#2 $\frac{5^{12}}{5^6}$ 5^6	$\frac{g^9}{g^3}$ g^6	$\frac{3^6 \cdot x^{11}}{3^2 \cdot x^7}$ $3^4 x^4$	$\frac{d^3 e^{10}}{d^2 e^6}$ de^4
R#3 $\frac{h^{11}}{h^3}$ h^8	$\frac{x^7 \cdot y^{16}}{x^4 \cdot y^{13}}$ $x^3 y^3$	$\frac{m^9 \cdot n^{12}}{m^2 \cdot n^{11}}$ $m^7 n$	$\frac{p^2 q^{16}}{p^2 q^{13}}$ q^3

Lesson 1.3 – Numbers in Exponential Form Raised to a Power

Recall that when we combine bases that are being multiplied we add the exponents. When we combine bases that are being divided we subtract the exponents. At the beginning of the last lesson we reviewed the rule for multiplying like bases in depth, let's take a closer look at our rule for dividing like bases.

In general, if *y* is nonzero and *m, n* are positive integers, then

$$\frac{y^m}{y^n} = y^{m-n} \qquad \text{if } m > n$$

Questions for discussion

- Why did the rule mention that *y* is "nonzero"?
- Why did they state that $m > n$?

____

Numbers in Exponential Form Raised to a Power

$$\left(7^4\right)^6$$

A number in exponential form raised to a power.

We will use the same strategy that we used in Lesson 1 and 2 to explore what occurs when a number in exponential form is raised to a power. We will expand our expression, condense it, then look for a pattern. However, we will not expand it completely. We will use the rule we discovered in lesson 1 to help save some time.

Complete the notes on the next page.

Class Notes – Expand the expression, then condense it.

LP#1 $(14^5)^3$ 14^{15}	$(9^7)^4$ 9^{28}
LP#2 $(4.3^3)^6$ 4.3^{18}	$(1.7^6)^5$ 1.7^{30}
LP#3 $((-7)^2)^5$ $(-7)^{10}$ If we were to express our exponential expression in standard form, would our answer be negative or positive? positive	$((-2.3)^7)^3$ $(-2.3)^{21}$ If we were to express our exponential expression in standard form, would our answer be negative or positive? negative

Use what you observe above to complete the following.

$$(x^a)^b = x^{a \cdot b}$$

When we raise a power to a power the exponent for the new expression is the (sum/difference/product/quotient) product of the original exponents.

Class Notes – Simplify the following expressions. Show your work by using one of the two methods below.

Examples of how to show your work.

Expanding and Condensing	Using the algorithm
$(3^8)^4 = 3^8 \cdot 3^8 \cdot 3^8 \cdot 3^8$	$(3^8)^4 = 3^{8 \cdot 4}$
$= 3^{8+8+8+8}$	$= 3^{32}$
$= 3^{32}$	

LP#4 $(8^{15})^6$ 8^{80}	$(15^9)^{11}$ 15^{99}	$(8.2^5)^7$ 8.2^{35}	$(0.25^{17})^4$ 0.25^{68}
LP#5 $((-7)^9)^{13}$ $(-7)^{117}$ If we were to express our exponential expression in standard form, would our answer be negative or positive? negative	$((-19)^5)^{20}$ $(-19)^{100}$ If we were to express our exponential expression in standard form, would our answer be negative or positive? positive	Kyle wrote $(5^4)^6 = 5^{10}$. Is he correct? If not, show work to correct his mistake. no, he is incorrect $(5^4)^6 = 5^{4 \cdot 6}$ $= 5^{24}$	

Review – Simplify the following expressions. Show work.

R#1 $(43^{11})^7$ 43^{77}	$(4.87^2)^{16}$ 4.87^{32}	Alicia is confused on whether to add or multiply the exponents when simplifying the following expression. $6^7 \cdot 6^{13}$ What does she need to do? She needs to add the exponents What answer should she get? 6^{20}
R#2 $(62^7)^7$ 62^{49}	$(0.125^4)^{12}$ 0.125^{48}	Simplify the following expression. Express your answer using a base of 9. $(81^3)^7$ 9^{42}
R#3 $((-17)^8)^5$ $(-17)^{40}$	$(0.8^6)^{30}$ 0.8^{180}	Simplify the following expression. Express your answer using a base of 2. $(16^2)^5$ 2^{40}

Lesson 1.4 – Numbers Raised to the Zeroth Power

Recall that when we raised an exponential expression to a power we multiplied the exponents. At the beginning of the last lesson we reviewed the rule for dividing like bases in depth, let's take a closer look at our rule for raising a power to a power.

For any number *y* and any positive integers *m* and *n*,

$$\left(y^m\right)^n = y^{mn}$$

because

$$\left(y^m\right)^n = (\underbrace{y \cdot y \cdots y}_{m \text{ times}})^n$$

$$=$$

$$= y^{mn}$$

Numbers in Exponential Form Raised to a Power of 0

5^0

A number in exponential form raised to the zeroth power.

We will use two methods of simplifying exponential expression to help determine what occurs when we raise a base to the zeroth power. In the first set of notes, we will expand our expression, condense it, then simplify. In the second set of notes, we will use the algorithm from lesson 2 for dividing numbers in exponential form.

Complete the notes on the next page.

Class Notes – Expand the expression, then condense it.

LP#1 $\frac{3^5}{3^5}$ 3^0 or 1	$\frac{5^4}{5^4}$ 5^0 or 1
LP#2 $\frac{x^7}{x^7}$ x^0 or 1	$\frac{y^{10}}{y^{10}}$ y^0 or 1
LP#3 $\frac{(-4)^3}{(-4)^3}$ $(-4)^0$ or 1	$\frac{(-2)^6}{(-2)^6}$ $(-2)^0$ or 1

Class Notes – Use the division rule and express your answers using powers.

LP#4 $\frac{3^5}{3^5}$ 3^0	$\frac{5^4}{5^4}$ 5^0
LP#5 $\frac{x^7}{x^7}$ x^0	$\frac{y^{10}}{y^{10}}$ y^0
LP#6 $\frac{(-4)^3}{(-4)^3}$ $(-4)^0$	$\frac{(-2)^6}{(-2)^6}$ $(-2)^0$

Use the two sets of class notes above to state the value of the exponential expressions below.

$3^0 = 1$	$5^0 = 1$	$x^0 = 1$	$y^0 = 1$	$(-4)^0 = 1$	$(-2)^0 = 1$

In general, what can we say that "anything" raised to the zeroth power is equivalent to?

$$(\text{anything})^0 = 1$$

Class Notes – Simplify the following expressions. When necessary, show your work.

LP#7 5^0 1	13^0 1	$(-11)^0$ 1	$(-13)^0$ 1
LP#8 y^0 1	m^0 1	$(3b)^0$ 1	$(-7g)^0$ 1
LP#9 $(xy)^0$ 1	xy^0 x	$3b^0$ 3	$-7g^0$ −7

Class Notes – Write the expanded expression in standard form.

LP#10 $(6 \times 10^4) + (8 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (5 \times 10^0)$	68,395
$(4 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + (9 \times 10^0)$	4,729

Class Notes – Write the following expression in expanded form using exponential notation.

LP#11 9,485	$(9 \times 10^3) + (4 \times 10^2) + (8 \times 10) + (5 \times 10^0)$
985,062	$(9 \times 10^5) + (8 \times 10^4) + (5 \times 10^3) + (6 \times 10^1) + (2 \times 10^0)$

Review – Simplify the following expressions.

R#1 8^0 1	10^0 1	$(-3)^0$ 1	$-(7)^0$ −1
R#2 2^0 1	w^0 1	$(-6)^0$ 1	$4x^0$ 4
R#3 z^0 1	$(-13)^0$ 1	$8m^0$ 8	$(-b)^0$ 1

Lesson 1.5 – Numbers Raised to a Negative Exponent

Recall that when we raise "anything" to the zeroth power, the value of the exponential expression is __1__. Technically, that is not entirely true. There is one value that cannot be raised to the a power of zero. Do you know what value it is?

For any number *y*, such that $y \neq 0$, $$y^0 = 1$$

$$3^{-4}$$

A number in exponential form raised to a negative exponent.

Numbers Raised to a Negative Exponent

Similar to the last lesson, we will use two methods of simplifying exponential expression to help determine what occurs when we raise a base to a negative power. In the first set of notes, we will expand our expression, condense it, then simplify. In the second set of notes, we will use the algorithm from lesson 2 for dividing numbers in exponential form.

Complete the notes on the next page.

Class Notes – Expand the expression, then condense it. Express your answer using powers.

LP#1 $\frac{2^3}{2^7}$ $\frac{1}{2^4}$	$\frac{6^3}{6^5}$ $\frac{1}{6^2}$
LP#2 $\frac{x^2}{x^7}$ $\frac{1}{x^5}$	$\frac{y^4}{y^{10}}$ $\frac{1}{y^6}$
LP#3 $\frac{14}{14^4}$ $\frac{1}{14^3}$	$\frac{w}{w^6}$ $\frac{1}{w^5}$

Class Notes – Use the division rule and express your answers as a power.

LP#4 $\frac{2^3}{2^7}$ 2^{-4}	$\frac{6^3}{6^5}$ 6^{-2}
LP#5 $\frac{x^2}{x^7}$ x^{-5}	$\frac{y^4}{y^{10}}$ y^{-6}
LP#6 $\frac{14}{14^4}$ 14^{-3}	$\frac{w}{w^6}$ w^{-5}

Use the two sets of class notes above to simplify the exponential expressions below.

$2^{-4} = \frac{1}{2^4}$	$6^{-2} = \frac{1}{6^2}$	$x^{-5} = \frac{1}{x^5}$	$y^{-6} = \frac{1}{y^6}$	$14^{-3} = \frac{1}{14^3}$	$w^{-5} = \frac{1}{w^5}$

Complete the rule for negative exponents below by reflecting on what occurred with our notes.

$$x^{-a} = \frac{1}{x^{a}}$$

Class Notes –Express the following expressions using positive exponents.

LP#7 5^{-3} $\frac{1}{5^3}$	13^{-2} $\frac{1}{13^2}$	2^{-4} $\frac{1}{2^4}$	3^{-3} $\frac{1}{3^3}$
LP#8 y^{-5} $\frac{1}{y^5}$	m^{-8} $\frac{1}{m^8}$	15^{-1} $\frac{1}{15}$	7^{-2} $\frac{1}{7^2}$
LP#9 x^{-10} $\frac{1}{x^{10}}$	m^{-1} $\frac{1}{m}$	9^{-2} $\frac{1}{9^2}$	4^{-3} $\frac{1}{4^3}$
LP#10 $(3m)^{-4}$ $\frac{1}{(3m)^4}$	$3m^{-4}$ $\frac{3}{m^4}$	$(4ab)^{-2}$ $\frac{1}{(4ab)^2}$	$4ab^{-2}$ $\frac{4a}{b^2}$

Review – Express the following expressions using positive exponents.

R#1 3^{-2} $\frac{1}{3^2}$	5^{-4} $\frac{1}{5^4}$	x^{-2} $\frac{1}{x^2}$	y^{-5} $\frac{1}{y^5}$
R#2 10^{-2} $\frac{1}{10^2}$	6^{-3} $\frac{1}{6^3}$	p^{-3} $\frac{1}{p^3}$	d^{-6} $\frac{1}{d^6}$
R#3 9^{-1} $\frac{1}{9}$	2^{-3} $\frac{1}{2^3}$	k^{-10} $\frac{1}{k^{10}}$	h^{-7} $\frac{1}{h^7}$

Laws of Exponents Activity (Assigned after lesson 1.5)

Recall that when we raise a base to a negative exponent, we can convert it to an equivalent expression that contains a positive exponent.

For any positive number *y*, and for any positive integer *n*, we define $$y^{-n} = \frac{1}{y^n}$$

Checking Other Students' Work

For this activity, we will check other students' work. Unfortunately, these students are struggling to keep all of their rules for exponents straight. You are to…

- look over each students work
- find their mistake
- correct the step at which they made the mistake
- finish the problem showing all the correct work

Look at the examples below.

Problem #1	Correction #1	Problem #2	Correction #2
$5^8 \cdot 7^2 \cdot 5^4$		$2^5 \cdot 4^2$	
$5^8 \cdot 5^4 \cdot 7^2$		8^{5+2}	$2^5 \cdot (2^2)^2$
$\cancel{5^{8\times4} \cdot 7^2}$	$5^{8+4} \cdot 7^2$	8^7	$2^5 \cdot 2^4$
$\cancel{5^{32} \cdot 7^2}$	$5^{12} \cdot 7^2$		2^9

Problem #3	Correction #3	Problem #4	Correction #4
$6^4 \cdot 4^8 \cdot 4^3 \cdot 6^{12}$ $6^4 \cdot 6^{12} \cdot 4^8 \cdot 4^3$ $36^{4+12} \cdot 16^{8+3}$ $36^{16} \cdot 16^{11}$	$6^{16} \cdot 4^{11}$	$9^{12} \div 9^3$ $9^{12 \div 3}$ 9^4	9^9

Problem #5	Correction #5	Problem #6	Correction #6
$(14^8)^5$ 14^{8+5} 14^{13}	14^{40}	$(1.3^5 \times 1.3^2)^0$ $(1.3^{5+2})^0$ $(1.3^7)^0$ $1.3^{7\times 0}$ 13^0 0	1

Problem #7	Correction #7	Problem #8	Correction #8
2^{-3} $\frac{1}{2^3}$ $\frac{1}{6}$	$\frac{1}{8}$	3^{-4} $(-3)^4$ $(-3)\cdot(-3)\cdot(-3)\cdot(-3)$ 81	$\frac{1}{81}$

Problem #9	Correction #9	Problem #10	Correction #10
$\frac{2^{17} \times 2^{-5}}{2^6}$ $\frac{2^{17+(-5)}}{2^6}$ $\frac{2^{12}}{2^6}$ $2^{12 \div 6}$ 2^2 4	2^6	$\frac{3^8 \times 4^3}{4^5 \times 3^5}$ $\frac{3^8 \times 4^3}{3^5 \times 4^5}$ $3^{8-5} \times 4^{3-5}$ $3^3 \times 4^{-2}$ $27 \times (-16)$ -432	$\frac{3^3}{4^2}$