Lesson 1.6 – Multiplying a Single Digit by a Power of 10 (+ Exponents)

Recall that every digit in a number is associated with a place value. Let's review the place values of the following number. Fill in the boxes below using the appropriate terms from the word bank.

7 2 , 4 8 5 . 3 9 1 6

ten-thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths

WORD BANK				
tenths	ten-thousands	hundredths	tens	ten-thousandths
hundreds	thousandths	ones	thousands	

Multiplying a Single Digit by a Power of 10 (positive exponents)

$$7 \times 10^5$$

A single digit being multiplied by a power of 10 containing a positive exponent.

When dealing with large numbers it can be helpful to express them using a power of ten. To begin, we will focus on powers of ten that contain a positive exponent. Place values that are associated with these powers of ten are found on the left side of the decimal. We will complete an activity that just focuses on these digits. We will take a closer look at the digits on the right side of the decimal in our next lesson.

Complete the activity on the next page.

Activity 1 - Express each number on the left as a power of ten. You may use the "Power of Ten Bank" for help.

$1 = 10^0$	**Power of Ten Bank**
$1{,}000{,}000 = 10^6$	10^3 10^9
$100 = 10^2$	10^4 10^7
$1{,}000{,}000{,}000 = 10^9$	10^8 10^1
$100{,}000{,}000 = 10^8$	10^6 10^5 10^0
$10{,}000 = 10^4$	10^2
$10 = 10^1$	
$10{,}000{,}000 = 10^7$	
$100{,}000 = 10^5$	
$1{,}000 = 10^3$	

Explain the pattern that you see above.

Activity 2 – Fill in the chart below. Use the example as a guide.

7 2 , 4 8 5 . 3 9 1 6

Numerical Description for place value	10,000	1,000	100	10	1				
Description in the base of 10	10^4	10^3	10^2	10^1	10^0				
Representing the digit using the base of 10	7×10^4	2×10^3	4×10^2	8×10	5×10^0				

Activity 3 – Determine the value for the "x" that would make the statement true.

$5\times10^x=5{,}000$	$6\times10^x=600{,}000$	$9\times10^x=90{,}000$	$3\times10^x=300$	$2\times10^x=2{,}000{,}000$
3	5	4	2	6

Explain how you determined each "x" in Activity 3.

Class Notes – Simplify each of the following.

LP#1 $7\times10^4=$ 70,000	$5\times10^6=$ 5,000,000	$8\times10^2=$ 800
LP#2 $3\times10^{10}=$ 30,000,000,000	$6\times10^1=$ 60	$2\times10^8=$ 200,000,000
LP#3 $4\times10^0=$ 4	$9\times10^3=$ 9,000	$1\times10^5=$ 100,000

Class Notes – Write each number as a product of a whole number and a power of 10.

LP#4 2,000,000 2×10^6	6,000 6×10^3	90 9×10^1
LP#5 70,000 7×10^4	500,000 5×10^5	30,000,000 3×10^7
LP#6 400 4×10^2	8,000,000 8×10^6	2,000 2×10^3

More Practice.

1) What power of 10 is found in between 2,321 and 456?

10^3

2) What power of 10 is found in between 879,432 and 2,045,565?

10^6

3) What power of 10 is equivalent to 1,000,000,000?

10^9

4) Rewrite $(5\times10^4)+(9\times10^2)+(7\times10^1)\times(3\times10^0)$ in standard form.

50,973

Review – In the **left column** simplify each expression. In the **right column** write each number as a product of a whole number and a power of 10.

R#1 $7 \times 10^9 =$ 7,000,000,000 $4 \times 10^0 =$ 4	200 2×10^2 5,000,000 5×10^6
R#2 $9 \times 10^{11} =$ 900,000,000,000 $2 \times 10^4 =$ 20,000	9 9×10^0 7,000,000,000 7×10^9
R#3 $6 \times 10^7 =$ 60,000,000 $3 \times 10^0 =$ 3	3,000 3×10^3 80,000 8×10^4

Lesson 1.7 – Multiplying a Single Digit by a Power of 10 (- exponents)

Recall that in the last lesson we explored the powers of ten that are on the left side of the decimal. Let's compare powers of ten that are on the left side of the decimal with their counterparts that are found on the right side.

Ten vs. Tenth		Hundred vs. Hundredth		Thousand vs. Thousandth		Ten-Thousand vs. Ten-Thousandth	
10	0.1	100	0.01	1000	0.001	10,000	0.0001

Hundred-Thousand vs. Hundred-Thousandth		Million vs. Millionth	
100,000	0.00001	1,000,000	0.000001

Discuss how the powers and their counter parts are alike. How are they different?

Multiplying a Single Digit by a Power of 10 (negative exponents)

$$4 \times 10^{-8}$$

A single digit being multiplied by a power of 10 containing a negative exponent.

When dealing with small numbers it can be helpful to express them using a power of ten. In order to do so, we will focus on powers of ten that contain a negative exponent. Place values that are associated with these powers of ten are found on the right side of the decimal. We will complete an activity that just focuses on these digits.

Activity 1 - Express each number on the left as a power of ten and a decimal. You may use the "Answer Bank" for help.

Fraction	Decimal		Power
$\frac{1}{10}$ =	0.1	=	10^{-1}
$\frac{1}{100,000}$ =	0.00001	=	10^{-5}
$\frac{1}{100}$ =	0.01	=	10^{-2}
$\frac{1}{10,000}$ =	0.0001	=	10^{-4}
$\frac{1}{1,000}$ =	0.001	=	10^{-3}

Answer Bank

0.00001

10^{-3}

10^{-5}

10^{-4} 0.001

0.0001 10^{-1}

0.1 10^{-2} 0.01

Activity 2 - Calculate the following, then express your answer as a whole number multiplied to a power of 10.

0.0001 x 9 0.0009 9×10^{-4}	0.01 x 4 0.04 4×10^{-2}
0.000001 x 7 0.000007 7×10^{-6}	0.00000001 x 5 0.00000005 5×10^{-8}

Activity 3 – Determine the value for the "?" that would make the statement true.

$3\times10^{?}=0.0003$	$5\times10^{?}=0.005$	$8\times10^{?}=0.8$	$6\times10^{?}=0.00006$	$2\times10^{?}=0.0000002$
-4	-3	-1	-5	-7

Explain how you determined each "?" in Activity 3.

Class Notes – Simplify each of the following.

LP#1 $7\times10^{-4}=$ 0.0007	$5\times10^{-6}=$ 0.000005	$8\times10^{-2}=$ 0.08
LP#2 $3\times10^{-10}=$ 0.0000000003	$6\times10^{-1}=$ 0.6	$2\times10^{-8}=$ 0.00000002
LP#3 $4\times10^{0}=$ 4	$9\times10^{-3}=$ 0.009	$1\times10^{-5}=$ 0.00001

Class Notes – Write each number as a product of a whole number and a power of 10.

LP#4 0.00002 2×10^{-5}	0.6 6×10^{-1}	0.009 9×10^{-3}
LP#5 0.0007 7×10^{-4}	0.00005 5×10^{-5}	0.00000003 3×10^{-8}
LP#6 0.04 4×10^{-2}	0.000008 8×10^{-6}	0.002 2×10^{-3}

Review – In the **left column** simplify each expression. In the **right column** write each number as a product of a whole number and a power of 10.

R#1 $7 \times 10^{-9} =$ 0.000000007 $4 \times 10^{-1} =$ 0.4	0.002 2×10^{-3} 0.0000005 5×10^{-7}
R#2 $9 \times 10^{-11} =$ 0.00000000009 $2 \times 10^{-4} =$ 0.0002	0.9 9×10^{-1} 0.0000000007 7×10^{-10}
R#3 $6 \times 10^{-7} =$ 0.0000006 $3 \times 10^{-3} =$ 0.003	0.0003 3×10^{-4} 0.00008 8×10^{-5}

Lesson 1.8 – Scientific Notation

Recall that we can use powers of ten to describe place values. We also can use powers of ten to write standard numbers in an expanded form. Rewrite the numbers below in expanded form by using powers of ten.

567,213 $(5\times10^5)+(6\times10^4)+(7\times10^3)+(2\times10^2)+(1\times10^1)+(3\times10^0)$
0.062954 $(6\times10^{-2})+(2\times10^{-3})+(9\times10^{-4})+(5\times10^{-5})+(4\times10^{-6})$
45,980.57 $(4\times10^4)+(5\times10^3)+(9\times10^2)+(8\times10^1)+(5\times10^{-1})+(7\times10^{-2})$
789.3082 $(7\times10^2)+(8\times10^1)+(9\times10^0)+(3\times10^{-1})+(8\times10^{-3})+(2\times10^{-4})$

Scientific Notation

$$8.35\times10^{12}$$

A number expressed in scientific notation.

Go to http://en.wikipedia.org/wiki/Scientific_notation . Read the introduction and the section titled "Normalized Notation", then go onto the activities on the next page.

Using the expression $a \times 10^b$, where a is any real number and b is an integer, complete the following:

Activity 1 - *Circle all values that could be used for a in normalized scientific notation.*

6.28	314	-10.4	7.32	13
-4.98	5.12	44	-235	7.99
90	-3.00	897	5	102

Activity 2 - Circle all values that can be used for b in normalized scientific notation.

6.2	3	~~10~~	-7.32	-1
4	-5	4.2	2.35	17
9	3.4	8	5.09	102

Activity 3 - Circle all the expressions that are expressed in normalized scientific notation.

6.28×10^5	314×10^{-2}	-10.4×10^8	$7.32 \times 10^{2.5}$	$13 \times 10^{-0.8}$
-4.98×10^3	$5.12 \times 10^{0.09}$	44×10^7	-235×10^{-6}	7.99×10^{15}
90×10^{-2}	$-3.00 \times 10^{8.25}$	897×10^1	5×10^{-5}	102×10^2

Class Notes – Write each expression in decimal form.

LP#1 $7.00 \times 10^4 =$ 70,000	$5.00 \times 10^6 =$ 5,000,000	$8 \times 10^2 =$ 800
LP#2 $7.21 \times 10^4 =$ 72,100	$5.89 \times 10^6 =$ 5,890,000	$8.1 \times 10^2 =$ 810
LP#3 $7.00 \times 10^{-4} =$ 0.0007	$5.00 \times 10^{-6} =$ 0.000005	$8 \times 10^{-2} =$ 0.08
LP#4 $7.21 \times 10^{-4} =$ 0.000721	$5.89 \times 10^{-6} =$ 0.00000589	$8.1 \times 10^{-2} =$ 0.081

Explain the pattern that you see above in relation to exponents being positive.

Explain the pattern that you see above in relation to exponents being negative.

Class Notes – Express each number using scientific notation.

LP#5 534,000 5.34×10^5	6,500 6.5×10^3	985,000,000 9.85×10^8
LP#6 0.00083 8.3×10^{-4}	0.0000000121 1.21×10^{-8}	0.00005732 5.732×10^{-5}
LP#7 9,310,000,000 9.31×10^9	0.000000398 3.98×10^{-7}	443 4.43×10^2

Extra Problems

1) Earth is approximately 93 million miles away from the sun. Express this distance in scientific notation. $93,000,000 = 9.3 \times 10^7$
2) Jupiter is approximately 4.83×10^8 miles away from the sun. Express this distance in standard form.
   483,000,000

Review – In the **left column** write each expression in decimal form. In the **right column** express each decimal using scientific notation.

R#1 $6.90 \times 10^3 =$ 6,900 $4.68 \times 10^{-5} =$ 0.0000468	8,350 8.35×10^3 0.0432 4.32×10^{-2}
R#2 $7.01 \times 10^4 =$ 70,100 $2.56 \times 10^{-9} =$ 0.00000000256	9,210,000 9.21×10^6 0.000054 5.4×10^{-5}
R#3 $9.23 \times 10^5 =$ 923,000 $2.71 \times 10^{-7} =$ .000000271	360,000 3.6×10 0.000781 7.81×10^{-4}

Lesson 1.8

<u>Lesson 1.9 – Operations Using Numbers in Scientific Form (Multiplying/Dividing)</u>

Recall that a number in scientific notation is expressed in the form of $a \times 10^b$, where a is any real number and $1 \le a < 10$, and b is an integer. The numbers below are expressed in $a \times 10^b$ form, but need a little adjusting so that they are in proper scientific notation. Adjust the expressions so that they are in proper scientific notation.

12.34×10^6	435×10^{-9}	0.0357×10^{-11}	0.000062×10^{13}
1.234×10^7	4.35×10^{-7}	3.57×10^{-13}	6.2×10^8

Also, recall that the commutative property of multiplication allows us to interchange terms connected by multiplication. We can use this property strategically to handle expressions. Let's discuss the examples below.

5 x 3.45 x 2	$(6x^3)(5x^8)$	$(m^2 \times 10^5)(m^4 \times 10^8)$

<u>Operations Using Numbers in Scientific Notation</u>

$$(1.2 \times 10^8)(3.5 \times 10^{14})$$

Two scientific numbers that will be multiplied.

In order to properly multiply or divide scientific numbers we need to express our answer using proper scientific notation and strategically use the commutative property of multiplication.

Class Notes – Simplify and express your answer in scientific notation.

LP#1 $(7 \times 10^4) \times (3 \times 10^5) =$ 2.1×10^{10}	$(4 \times 10^2) \times (8 \times 10^7) =$ 3.2×10^{10}
LP#2 $(6.23 \times 10^5) \times (9.76 \times 10^3) =$ 6.08048×10^9	$(2.38 \times 10^{11}) \times (1.99 \times 10^{10}) =$ 4.7362×10^{21}
LP#3 $(2 \times 10^{10}) \div (8 \times 10^7) =$ 2.5×10^2	$(1 \times 10^{14}) \div (4 \times 10^8) =$ 2.5×10^5
LP#4 $(3.92 \times 10^7) \div (1.12 \times 10^{11}) =$ 3.5×10^{-4}	$(8.04 \times 10^4) \div (2.01 \times 10^{13}) =$ 4×10^{-9}
LP#5 $(0.00000072) \div (2.88 \times 10^5) =$ 2.5×10^{-12}	$(6{,}390{,}000{,}000) \times (4.26 \times 10^7) =$ 1.5×10^{16}

Use the planet mass table[1] below to complete the tasks.

Rank	Name	Mass (kg)
1	Sun	1.9891×10^{30}
2	Jupiter	1.8986×10^{27}
3	Saturn	5.6846×10^{26}
4	Neptune	10.243×10^{25}
5	Uranus	8.6810×10^{25}
6	Earth	5.9736×10^{24}
7	Venus	4.8685×10^{24}
8	Mars	6.4185×10^{23}
9	Mercury	3.3022×10^{23}
10	Moon	7.349×10^{22}
11	Pluto	1.25×10^{22}

<table>
<tr>
<td>LP#6
How many times greater is the mass of Jupiter compared to the mass of Earth? Express your answer using scientific notation.

3.178×10^{2}</td>
<td>LP#7
How many times greater is the mass of Earth compared to the mass of Pluto? Express your answer using scientific notation.

4.779×10^{2}</td>
</tr>
<tr>
<td>LP#8
What is the mass of Neptune expressed in proper scientific notation?

1.0243×10^{26}</td>
<td>LP#9
Multiply the mass of Mars to the mass of Venus and express the result in scientific notation.

3.125×10^{48}</td>
</tr>
</table>

Review – Simplify and express your answer in scientific notation.

R#1 $(5 \times 10^3) \times (1 \times 10^2) =$ 6×10^5	$(6 \times 10^9) \div (3 \times 10^5) =$ 2×10^4
R#2 $(9 \times 10^6) \times (5 \times 10^7) =$ 4.5×10^{14}	$(7 \times 10^{17}) \div (2 \times 10^9) =$ 3.5×10^8
R#3 $(4 \times 10^1) \times (6 \times 10^8) =$ 2.4×10^{10}	$(9 \times 10^2) \div (4 \times 10^6) =$ 2.25×10^{-4}

[1]The table used for this activity is from http://www.smartconversion.com/otherInfo/Mass_of_planets_and_the_Sun.aspx

Lesson 1.10 – Order of Magnitude

Recall that we can use the commutative property of multiplication to evaluate an expression that involves multiplying two numbers in scientific notation. Simplify the expressions below. Express your answer using proper scientific notation.

$(8.32\times10^{7})\times(5.01\times10^{11})=$	$(4.08\times10^{5})\div(1.02\times10^{9})=$

Before we can begin to add or subtract numbers in scientific notation, we must explore the magnitude of a number in scientific notation.

Order of Magnitude

$$4.3\times10^{8}$$

A number expressed in scientific notation.
Its order of magnitude is 8.

Before we begin to add/subtract numbers in scientific notation, we need to focus on the base of 10. As you know a number expressed in scientific notation is expressed in the form of $a\times10^{b}$, where a is any real number and $1\le a<10$, and b is an integer. If you focus on the lead term, a, the value for it has its limits. The lead term can be equal to 1 or must be between 1 and 10. The base of 10 is the part of the expression that truly controls its size. As the value of the exponent, b, increases the size of the number grows increasingly larger. As the value of the exponent, b, decreases the size of the number becomes increasingly smaller. By looking at the exponent we can get a feel for the **magnitude** (how large or how small) of a number that is expressed in scientific notation. The **order of magnitude** of a number is the number of powers of 10 contained in the number[1].

We will complete the activity on the next page to become more acquainted with the order of magnitude.

Activity [2] – Fill in the missing values for the "Power of Ten" and "Order of Magnitude" columns.

In words	**Decimal**	**Power of ten**	**Order of magnitude**
septillionth	0.000,000,000,000,000,000,000,001	10^{-24}	−24
sextillionth	0.000,000,000,000,000,000,001	10^{-21}	−21
quintillionth	0.000,000,000,000,000,001	10^{-18}	−18
quadrillionth	0.000,000,000,000,001	10^{-15}	−15
trillionth	0.000,000,000,001	10^{-12}	−12
billionth	0.000,000,001	10^{-9}	−9
millionth	0.000,001	10^{-6}	−6
thousandth	0.001	10^{-3}	−3
hundredth	0.01	10^{-2}	−2
tenth	0.1	10^{-1}	−1
one	1	10^{0}	0
ten	10	10^{1}	1
hundred	100	10^{2}	2
thousand	1,000	10^{3}	3
million	1,000,000	10^{6}	6
billion	1,000,000,000	10^{9}	9
trillion	1,000,000,000,000	10^{12}	12
quadrillion	1,000,000,000,000,000	10^{15}	15
quintillion	1,000,000,000,000,000,000	10^{18}	18
sextillion	1,000,000,000,000,000,000,000	10^{21}	21
septillion	1,000,000,000,000,000,000,000,000	10^{24}	24

Class Notes – State the magnitude of expressions below. Then determine the smallest power of 10 that will exceed it.

LP#1 8.31×10^{13} 13 10^{14}	5.332×10^{21} 21 10^{22}
LP#2 4.03×10^{-19} -19 10^{-18}	1.56×10^{-6} ~~10~~ -6 10^{-5}
LP#3 345,700,675,019 11 10^{12}	8,452,935,012,498,001 15 10^{16}
LP#4 0.0000231 -5 10^{-4}	0.00000000753 -9 10^{-8}

Class Notes – For the pairs of numbers below, use a base of 10 to approximate how many times larger the larger number is than the smaller number.

LP#5 8.31×10^{13} 5.3×10^{19} 10^{6}	4.75×10^{27} 4.06×10^{17} 10^{10}
LP#6 345,700,675,019 8,452,935,012,498,001 10^{4}	52,763,985,123 1,788 10^{7}
LP#7 0.00003346 0.00100234 10^{2}	0.000000869 0.000000000988 10^{3}

Review – State the magnitude of expressions below. Then determine the smallest power of 10 that will exceed it.

R#1 6.19×10^{19} 19 20	5.26×10^{-24} -24 -23
R#2 3.03×10^{32} 32 33	2,560,700,615,015 12 13
R#3 4.96×10^{-11} -11 -10	0.00000153 6 7

Footnotes

[1] The definition for order of magnitude was taken from Wikipedia.org

[2] The table for the activity was a table taken from Wikipedia.org and has been modified

Lesson 1.11 – Operations Using Numbers in Scientific Form (Adding/Subtracting)

Recall that a number in scientific notation is expressed in the form of $a \times 10^b$, where a is any real number and $1 \leq a < 10$, and b is an integer. In order to add/ subtract numbers expressed in scientific notation, we need to review what occurs when we add/subtract variables and how we handle their coefficients. Let's review...

$5x+3x$	$7a+4b-3a+8b$	$12y-3y+2y^2$

Adding and subtracting numbers expressed in scientific notation is similar to what we see in our work above. Let's move onto the lesson and see how this is done.

Adding/Subtracting Numbers in Scientific Notation

$$(6.5\times10^7)+(8.2\times10^7)$$

Two numbers expressed in scientific notation that will be added together.

To help us understand how to add or subtract numbers in scientific notation we will compare the two examples below.

Ex.1 $(6.5\times10^7)+(8.2\times10^7)$	Ex. 2 $6.5n+8.2n$
1.47×10^8	$14.7n$

The key to adding or subtracting numbers that are in *a* x 10^b form is the magnitudes of the numbers. If the magnitude of both numbers is the same, we can add the '*a*' values. If the magnitude of the numbers is different, then we will need to make some adjustments before we proceed. Let's complete some notes to make it a little more concrete for us to understand.

Class Notes – Simplify the following. Express your answers using proper scientific notation.

LP#1 $(1.3\times10^{7})+(5.36\times10^{7})$ 6.66×10^{7}	$(6.23\times10^{12})+(2.08\times10^{12})$ 8.31×10^{12}
LP#2 $(8.54\times10^{-15})-(6.97\times10^{-15})$ 1.57×10^{-15}	$(7.92\times10^{-21})+(1.56\times10^{-21})$ 6.36×10^{-21}
LP#3 $(7.21\times10^{32})+(4.98\times10^{32})$ 1.219×10^{33}	$(3.7\times10^{-16})+(9.8\times10^{-16})$ 1.35×10^{-15}
LP#4 $(7.42\times10^{-53})-(6.89\times10^{-53})$ 5.3×10^{-54}	$(4.5\times10^{17})-(3.781\times10^{17})$ 7.19×10^{16}
LP#5 $(1.324\times10^{16})+(6.81\times10^{17})$ 6.9424×10^{17}	$(6.51\times10^{21})+(2.12\times10^{20})$ 6.722×10^{21}
LP#6 $(3.98\times10^{25})-(8.21\times10^{23})$ 4.0621×10^{25}	$(9.876\times10^{-17})-(7.546\times10^{-19})$ 9.95146×10^{-17}

Review – Simplify the following. Express your answers using proper scientific notation.

R#1 $(3.35\times10^{9})+(4.21\times10^{9})$ 7.56×10^{9}	$(8.23\times10^{14})-(1.86\times10^{13})$ 8.416×10^{14}
R#2 $(5.21\times10^{-18})-(4.69\times10^{-18})$ 5.2×10^{-19}	$(6.81\times10^{-23})+(2.96\times10^{-21})$ 3.0281×10^{-21}
R#3 $(1.99\times10^{42})+(9.91\times10^{40})$ 2.0891×10^{42}	$(8.9\times10^{-18})+(7.8\times10^{-18})$ 1.67×10^{-17}