Lesson 2.8 – More Rotations and Reflections

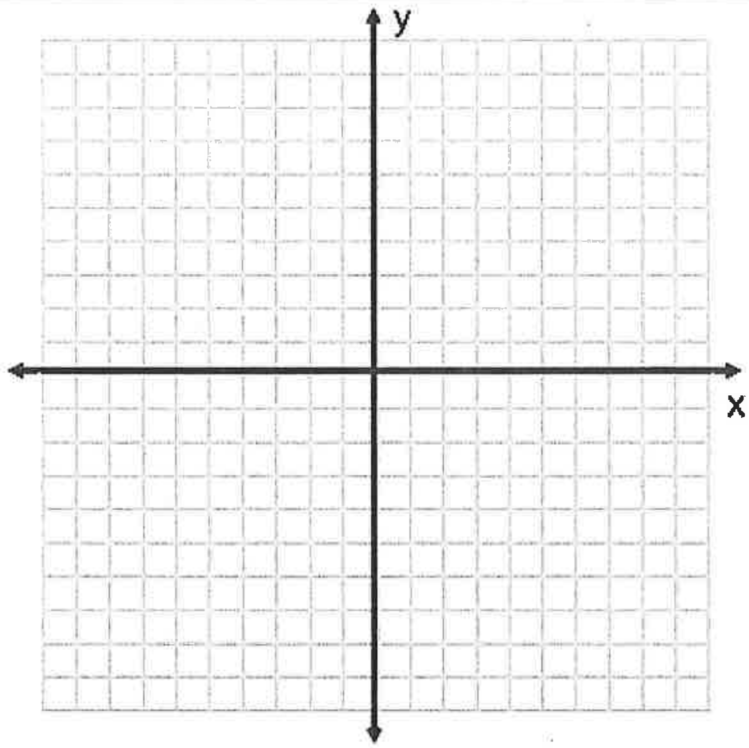
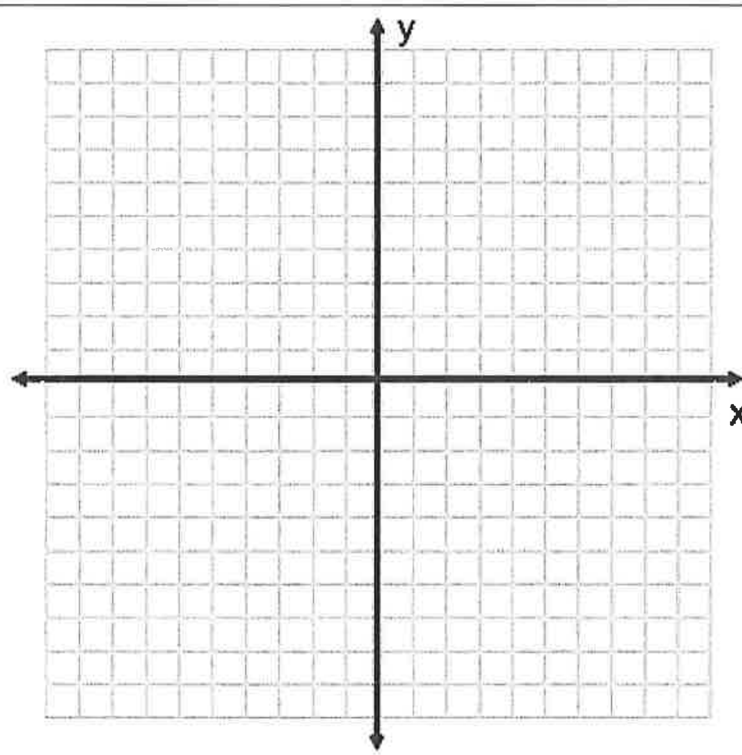
Before we conclude the first part of this module, we need to look into rotations and reflections more in-depth. So far we have rotated shapes about the ____________ and have reflected shapes through the ______________ and ______________. How do we handle rotations about a different point? How do we reflect through lines that are not axes?

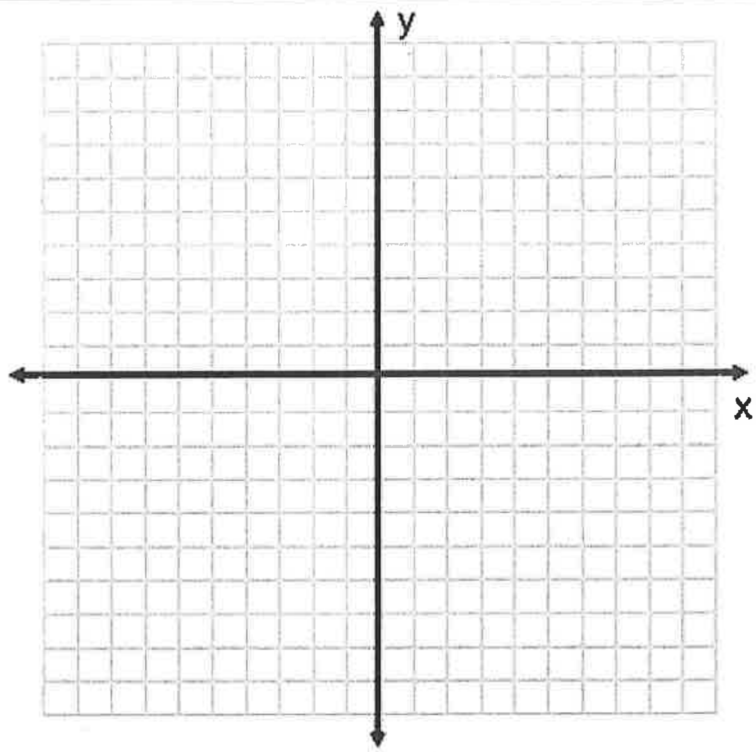
- Describe the process for rotating an object about a point on a "grid less" plane.

- Describe the process for rotating an object about the origin in the Cartesian plane.

- Describe the process for reflecting an object through a line.

Set 1 – Follow the instructions below.

A. Plot the points A(2, -8), B(6, -3) and C(9, -7). Connect the points to form triangle ABC. Rotate triangle ABC 90° about point (1, -4) and label the new image A'B'C'.	
B. Plot the points F(1, 3), G(1, 5), H(4 , 3) and I(4, 6). Connect the points to form quadrilateral FGHI. Rotate quadrilateral FGHI 90° about point (-2, 2) and label the new image F'G'H'I'.	

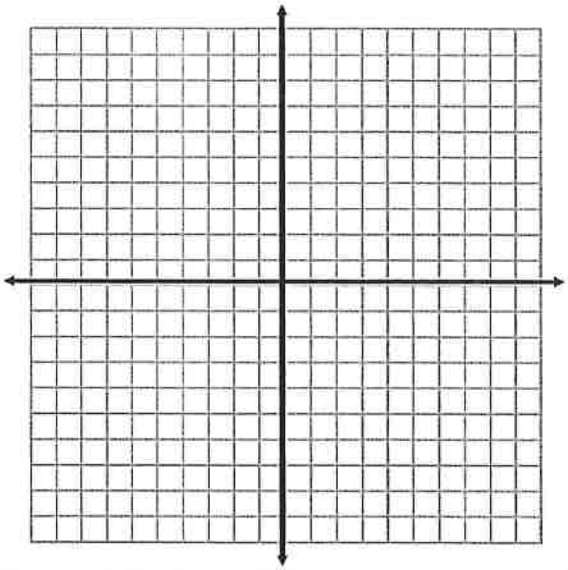
C. Plot the points S(0,3), T(4,0) and U(5,4). Connect the points to form triangle STU. Rotate triangle STU 180^{0} about point (3, -2) and label the new image S'T'U'.	

Set 2 – Use the instructions below and the graphs from **Set 1** to complete this set.

For Set 1 – A	For Set 1 – B	For Set 1 – C
• Draw a vertical, dotted line that passes through the point (-1, 0). • Reflect triangle A'B'C' through the dotted vertical line and label the image appropriately.	• Draw a horizontal, dotted line that passes through the point (0, -2). • Reflect quadrilateral F'G'H'I' through the dotted horizontal line and label the image appropriately.	• Draw a vertical, dotted line through point T'. • Reflect triangle triangle S'T'U' through the vertical line. Label both images appropriately.

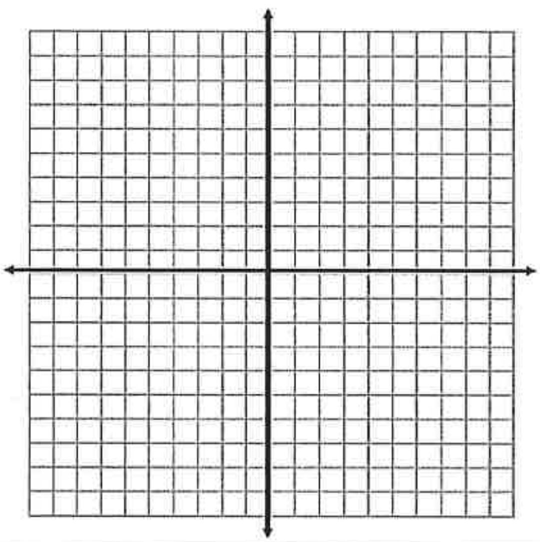
Questions for discussion

1. Refer back to Module 2-Lesson 6. Is there a single transformation that can undo a sequencing of two translations? Explain.
2. Refer back to Module 2-Lesson 7. Is there a single transformation that can undo a sequencing of a translation and a reflection? Explain.
3. Refer to this lesson. Is there a single transformation that can undo a rotation and a reflection? Explain.



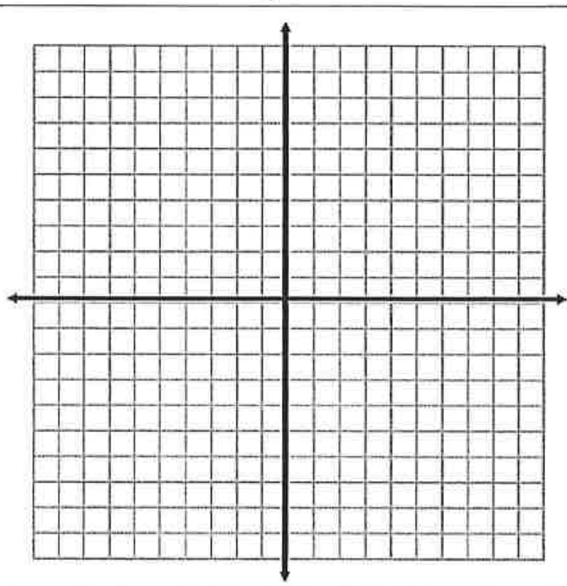
R#1

Label the axes. Plot the following points, connect the points to form triangle DEF: D(-4, -1) , E (-1, -1) , F(-2, -6). Rotate DEF -90^{0} about the origin and label the new image D'E'F'. Then, reflect D'E'F' over the vertical line that pass through the point (2, 3). Label the new image D"E"F".



R#2

Label the axes. Plot the following points, connect the points to form triangle XYZ: X(5, -2) , Y (2, 0) ,Z(1, -6). Reflect XYZ over the vertical line that passes through the point X and label it X'Y'Z'. Then rotate the triangle about the point (1, 2) 180^{0} and label the new image X"Y"Z".



R#3

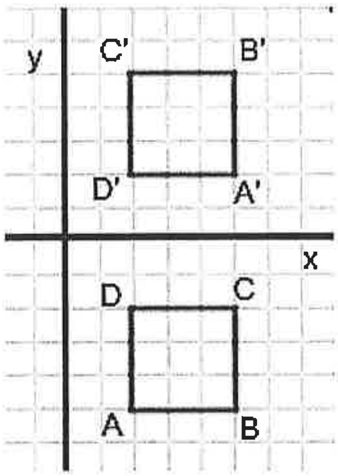
Label the axes. Plot the following points, connect the points to form quadrilateral ABCD: A(2, 2) , B(6, 2) , C(8, 6) , D(2, 6). Reflect quadrilateral ABCD over the horizontal line that passes through the point (-5, -2). Label the new image A'B'C'D'. Then, rotate A'B'C'D' about the point (0, -5) at an angle of 180^{0} and label the new image A"B"C"D".

Lesson 2.9 – Recognizing Sequences of Basic Rigid Motions

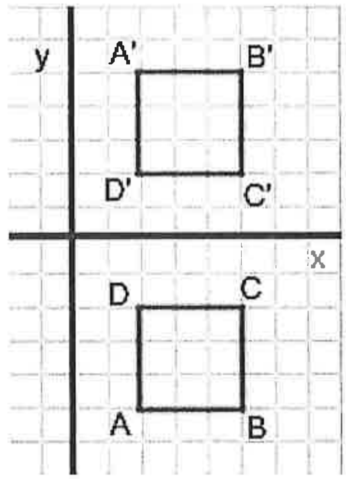
To conclude the first part of this module, we will look at two images and determine which rigid motion(s) would map the original shape to its 'prime' image. Before we begin, answer the questions below.

1) List the three basic rigid motions and list what is needed to perform each motion.
   -
   -
   -
2) Which of the three basic rigid motions changes the size of an object? Which of the three basic rigid motions changes the angle measures of objects?
3) Which of the three basic motions can change the 'order' in which the vertices of a shape are labeled from a clockwise direction to a counter-clockwise direction?
4) Angles rotated in a clockwise direction have what type of angle measure? Angles rotated in a counter-clockwise direction have what type of angle measure?
5) Name a positive angle measure and a negative angle measure that would take a shape to the same destination.

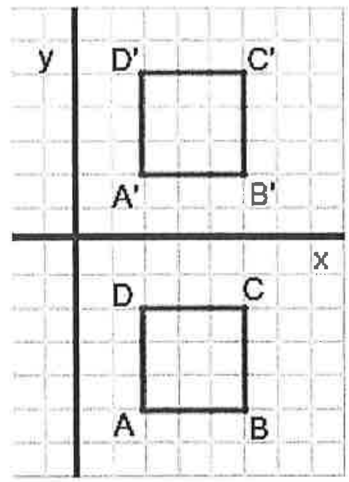
Set 1 - In the diagrams below, square ABCD has been transformed from quadrant IV to its new image in quadrant I. Even though the diagrams appear to look the same, a different rigid motion has been used for each. Determine the basic rigid motion used for each and explain your reasoning. Also state the following: 1) for the translation, describe the vector that was used, 2) for the reflection, state the line the object was reflect through, and 3) for the rotation, state the angle the object was rotated.



Rotation



Reflection



Translation

Answer these questions.

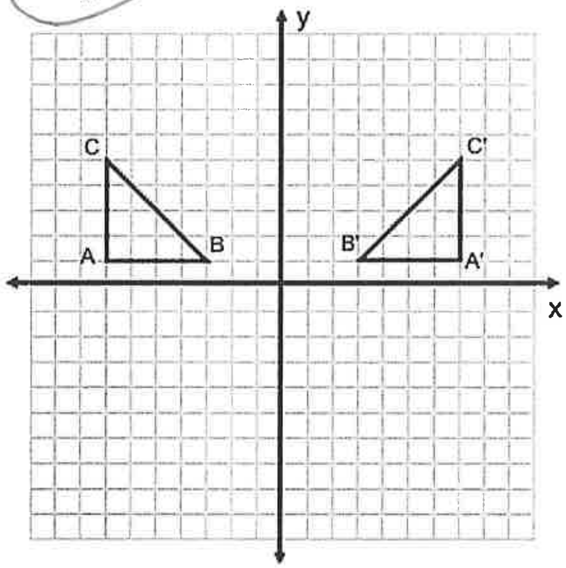
1. In the diagrams above, did any of the motions change the size of the shapes?

2. Did any of the motions change any of the angle measures?

Your answers to both questions should have been 'no'. Sliding, flipping, or turning an object does not change the measures of its sides or angles. Whenever we can use a sequence of rigid motions to map an object directly onto another object, both objects are said to be **congruent**. **Congruent** objects have the same shape and the same size.

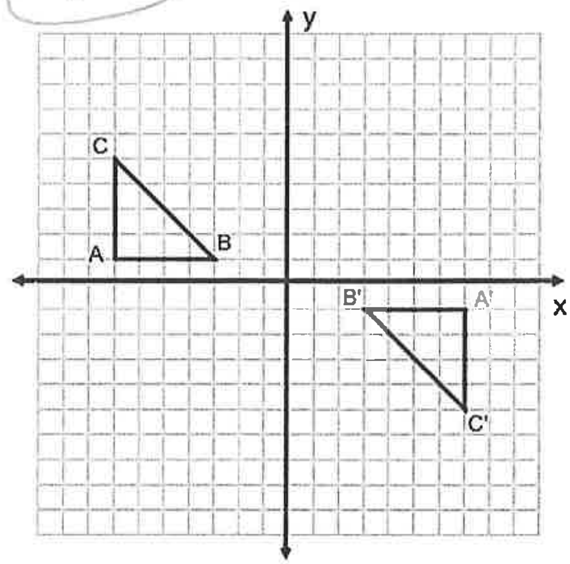
Set 2 – Circle whether the pair of objects are congruent or not congruent. If they are congruent, identify the basic rigid motion(s) one would need to use to map the original image to the 'prime' image

a) Congruent Not Congruent



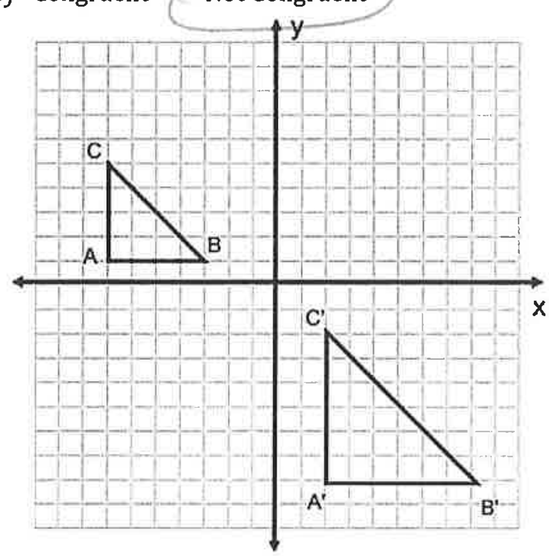
reflection

b) Congruent Not Congruent

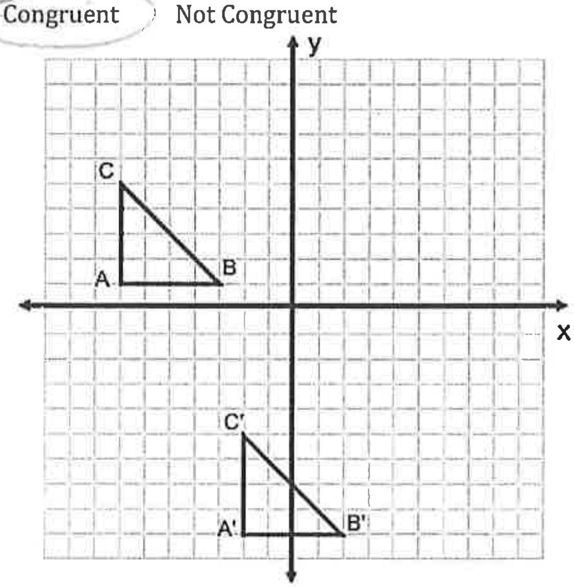


rotation

c) Congruent Not Congruent

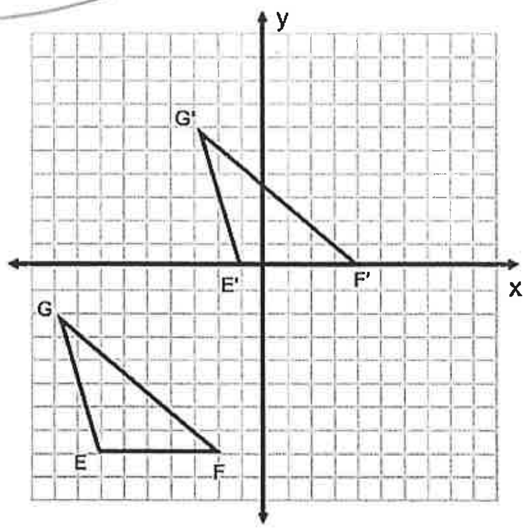


d) Congruent Not Congruent



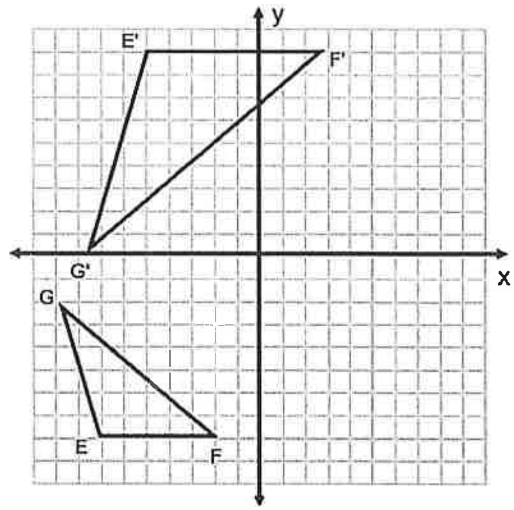
translation

e) Congruent Not Congruent

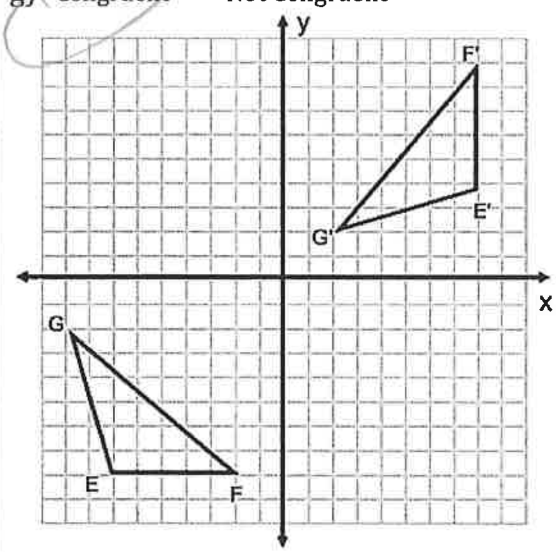


translation

f) Congruent Not Congruent

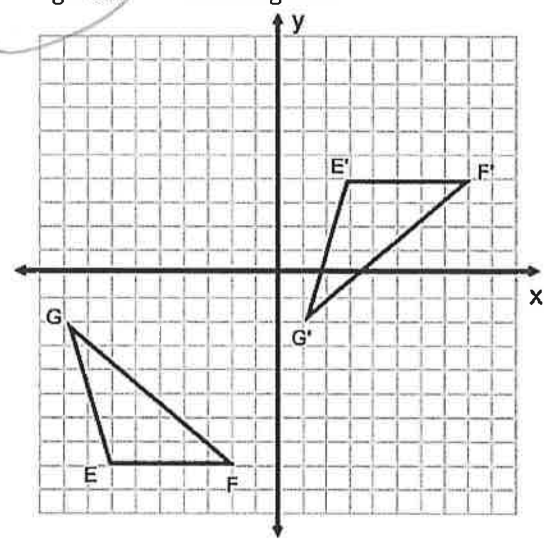


g) Congruent Not Congruent



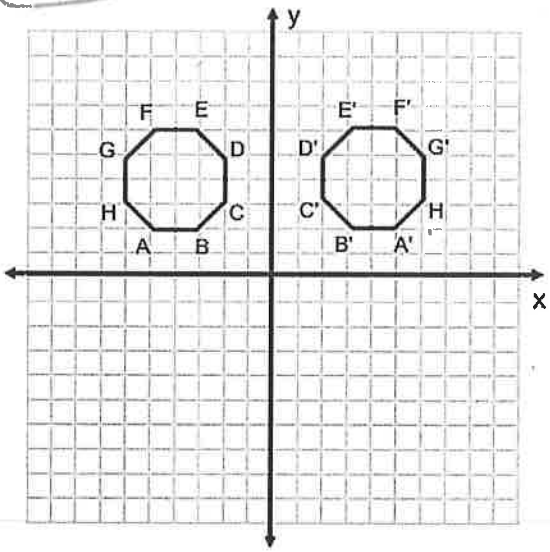
rotation
&
translation

h) Congruent Not Congruent



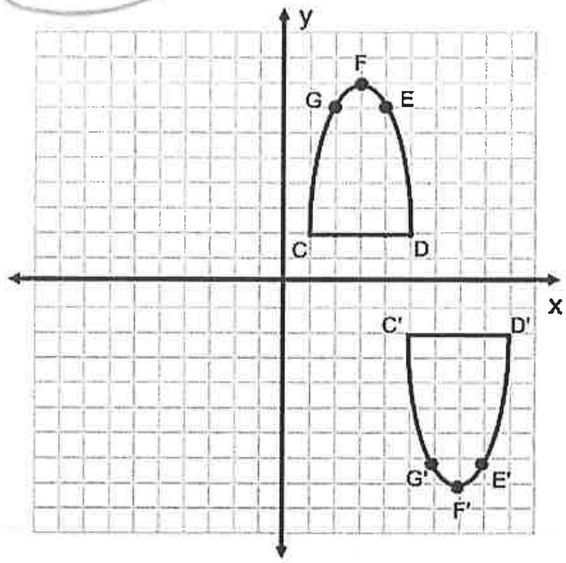
reflection
&
translation

i) Congruent Not Congruent



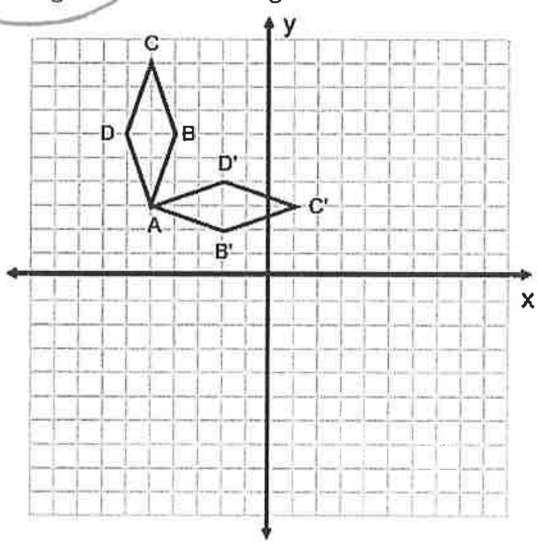
reflection

j) Congruent Not Congruent



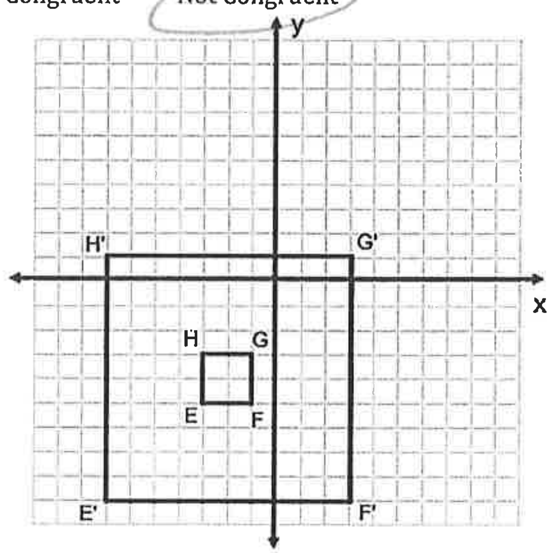
reflection
&
translation

k) Congruent Not Congruent



rotation

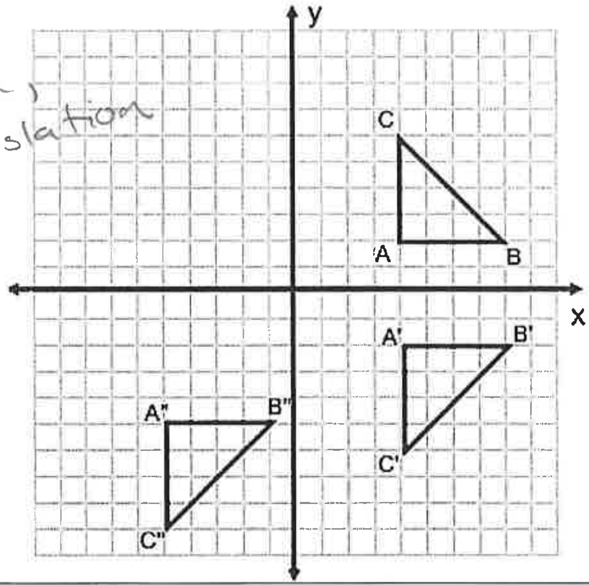
l) Congruent Not Congruent

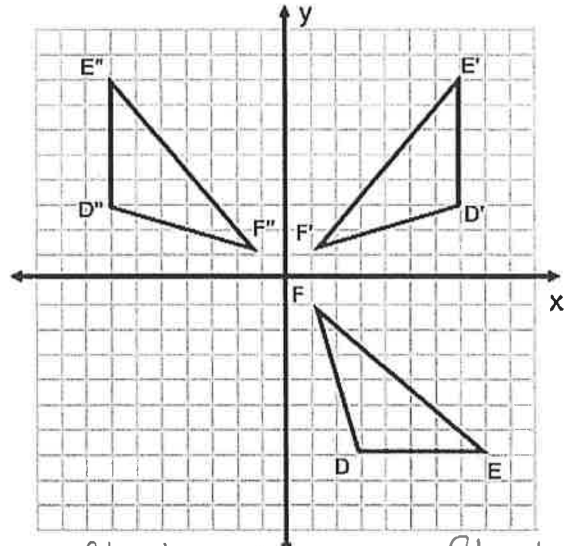


REVIEW – State which rigid motions are used to move the original shape to the "double prime" shape.

R#1

reflection, then translation

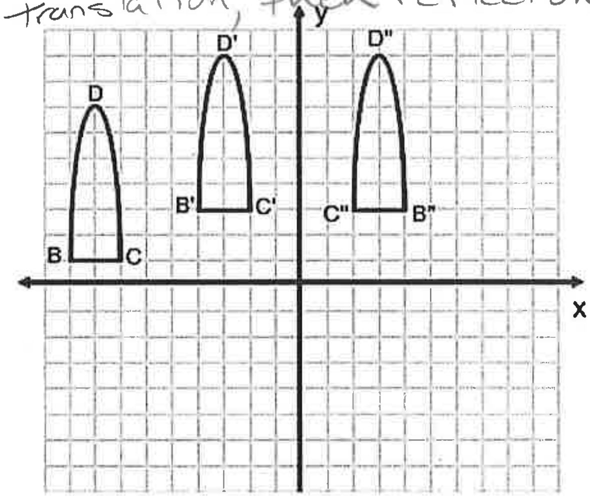


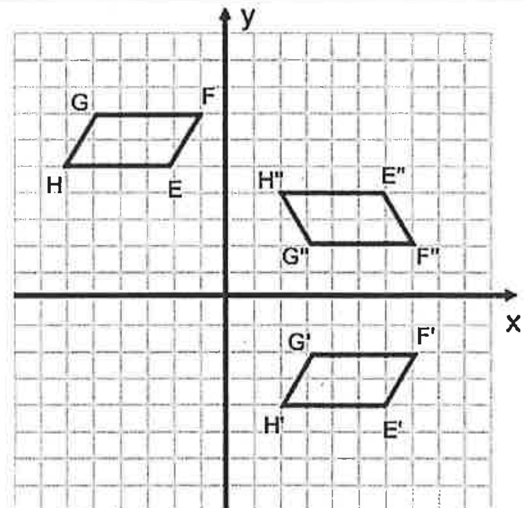


reflection, then reflection

R#2

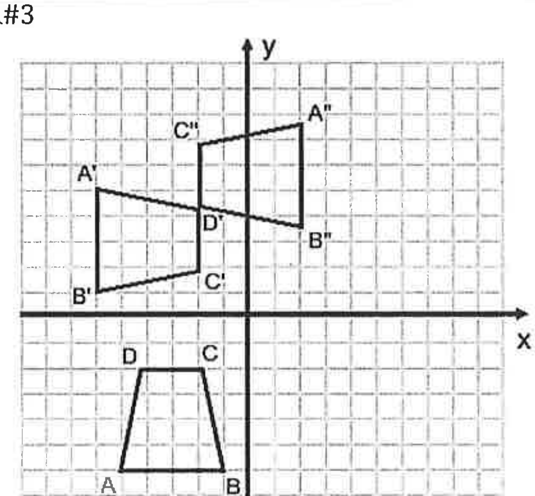
translation, then reflection



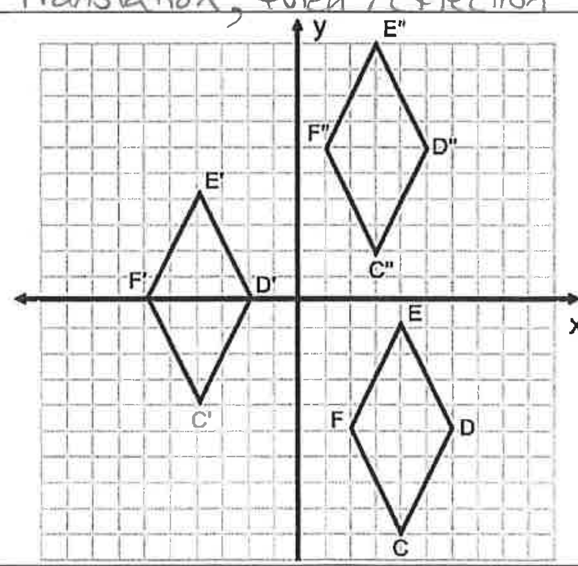


translation, then reflection

R#3



rotation, then reflection



translation, then translation

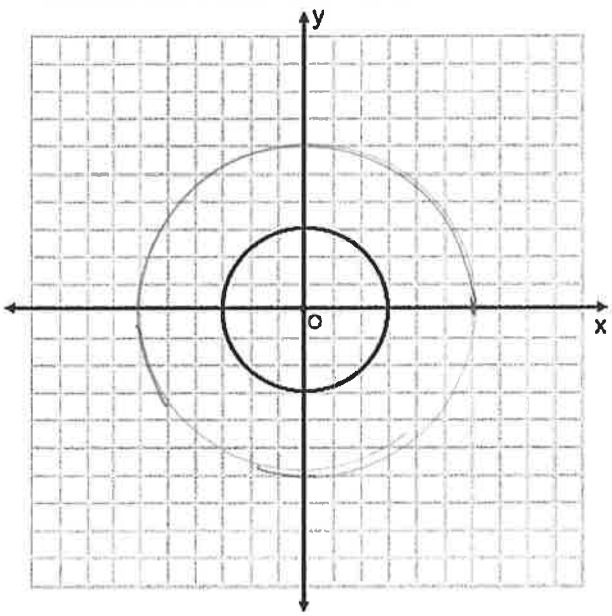
Lesson 2.10 – It's all in the eyes – Intro to Dilations

The pupil is a hole in the iris of your eye that controls the amount of light that enters your retina. The pupil will appear smaller in bright situations and the pupil will appear larger in dark situations. When pupils appear smaller than normal, they are said to be constricted. When pupils appear larger than normal they are said to be **dilated.** The concept of dilation in mathematics is similar to what occurs with your eyes, except when we dilate a shape, the **dilation** can make the shape larger or smaller. To fully understand the concept we will complete some activities.

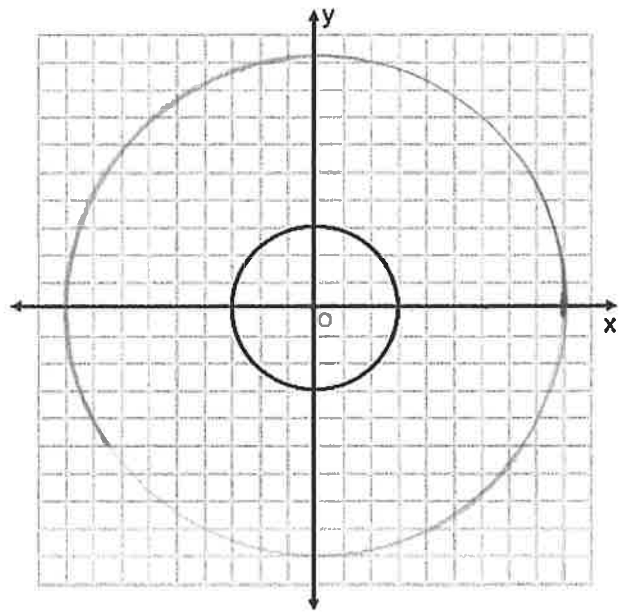
To perform dilations, we need to know the **scale factor** to be used. A scale factor is a number which enlarges or shrinks some quantity through multiplication.

Set 1 – In the following diagrams, point O is located at the origin and is the center of the circle. Given the scale factor, perform a dilation centered at O.

A) Dilate the circle using O as the center of dilation and a scale factor of 2.

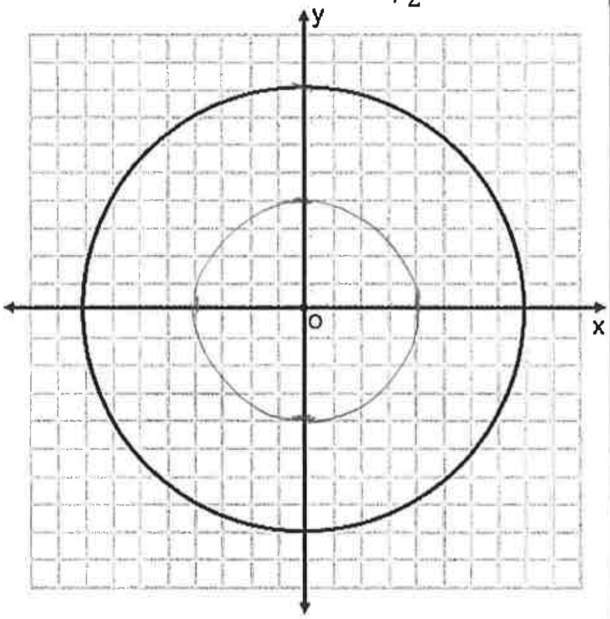


B) Dilate the circle using O as the center of dilation and a scale factor of 3.

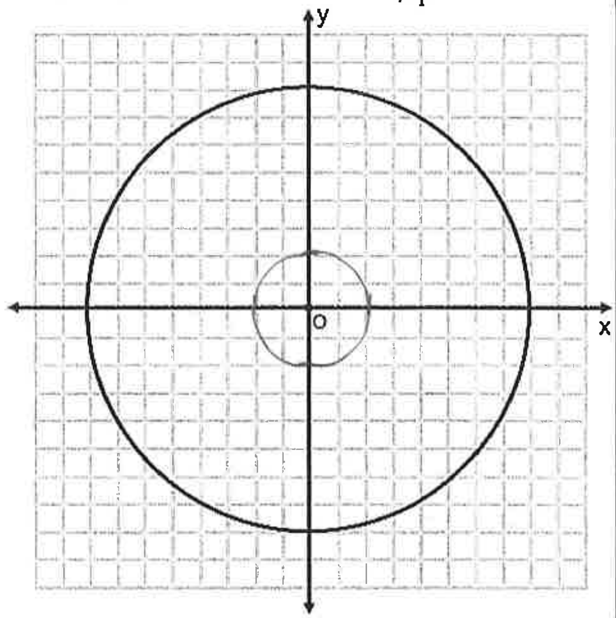


C) Imagine that the circles in the above exercises are pupils. Reflect on the reading and state a situation in which the pupils would exhibit the behavior of enlarging.

D) Dilate the circle using O as the center of dilation and a scale factor of $1/2$.

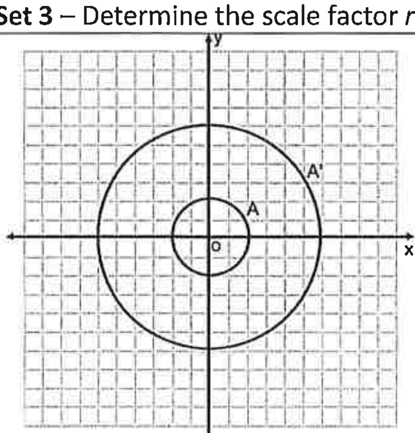


E) Dilate the circle using O as the center of dilation and a scale factor of $1/4$.

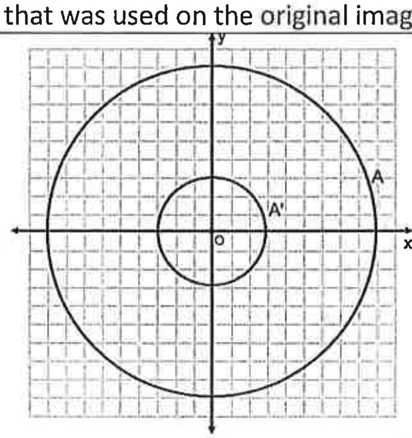


F) Imagine that the circles in the above exercises are pupils. Reflect on the reading and state a situation in which the pupils would exhibit the behavior of shrinking.

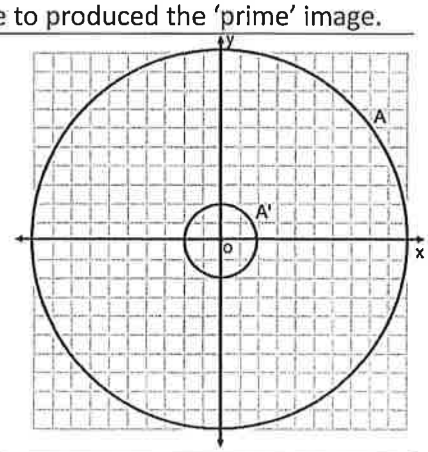
Set 3 – Determine the scale factor *r* that was used on the original image to produced the 'prime' image.



$K = K$



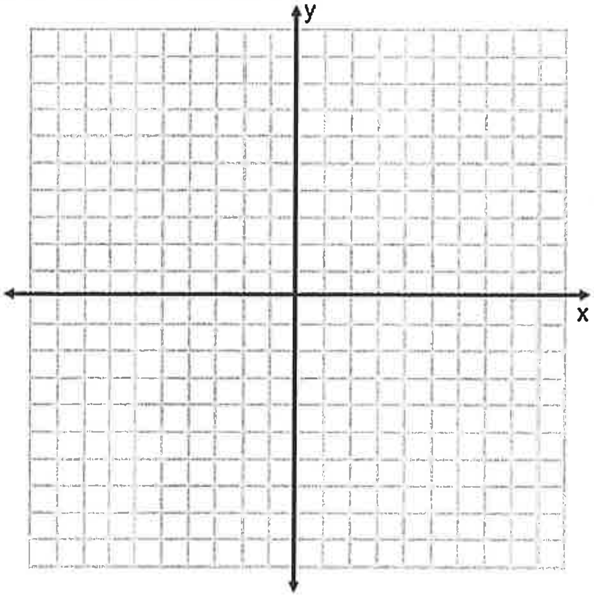
$K = 1/3$



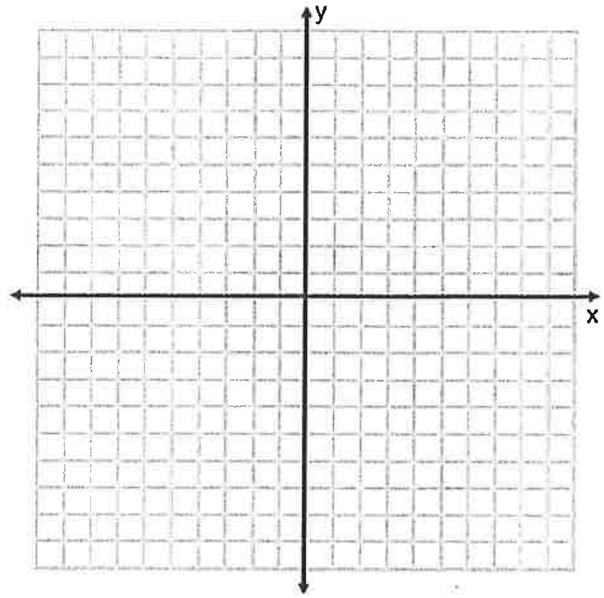
$K = \frac{1}{5}$

Review – Follow the instructions for each problem.

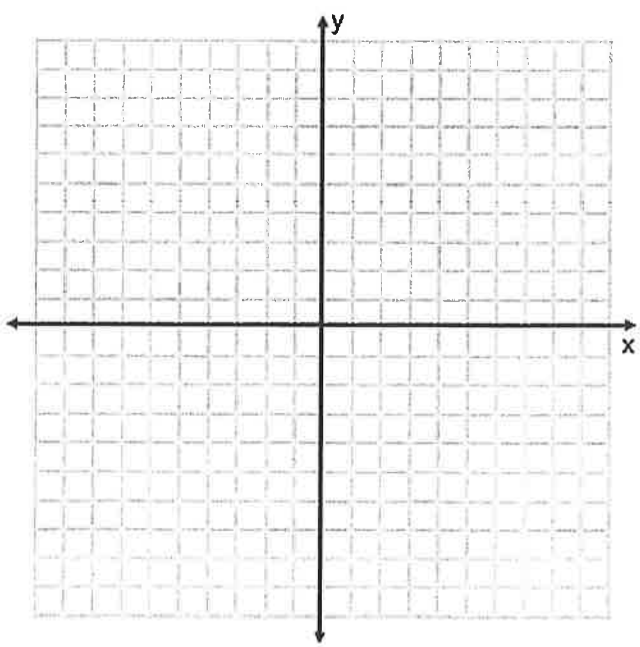
R#1 – Using a compass, draw a circle containing a radius of 4 and is centered at the origin. Label it B. Dilate the circle using a scale factor of 1.75 and draw. Label the image B'.



R#2 – Using a compass, draw a circle containing a radius of 10 and is centered at the origin. Label it C. Dilate the circle using a scale factor of 0.7 and draw. Label the image C'.



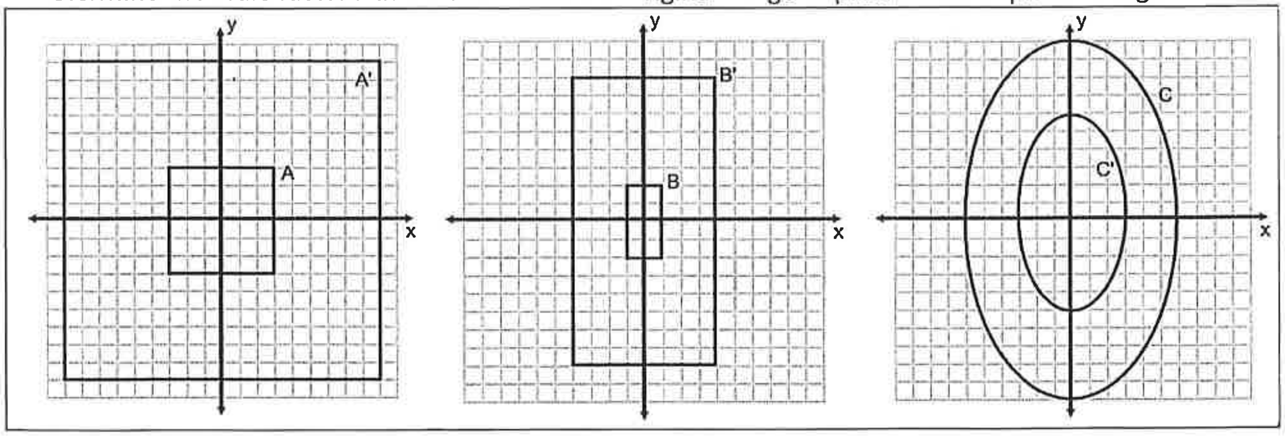
R#3 – Using a compass, draw a circle containing a radius of 6 and is centered at the origin. Label it D. Dilate the circle using a scale factor of 1.5 and draw. Label the image D'.



Lesson 2.11 – Dilations

Recall that the scale factor is a number that changes the size of an object through multiplication.

Determine the scale factor r that was used on the original image to produced the 'prime' image.



$r = 3$ $r = 4$ $r = 1/2$

Properties of Dilations

A center of dilation and a scale factor are needed to perform a dilation. To explore properties of dilations we will use a compass, a ruler, and a protractor. Complete the activities on the next page.

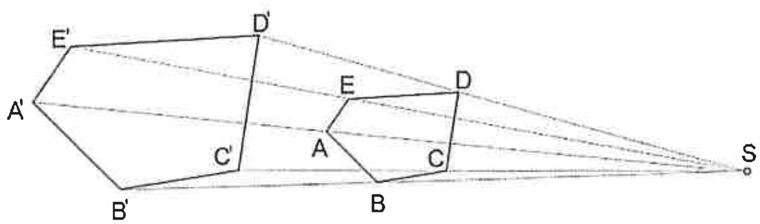
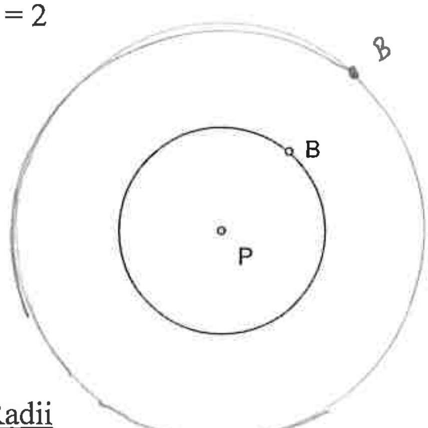


Figure ABCDE was dilated from center S and used a scale factor greater than 1 to produce figure A'B'C'D'E'.

Use point P as the center of dilation. Dilate each circle using the given scale factor, k. Use a compass and a straightedge to perform the dilations. Answer all questions.

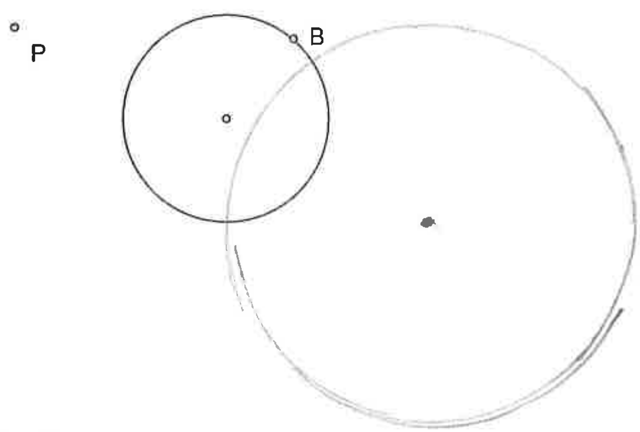
Set 1

$k = 2$



Radii
pre-image = 1/2 in image = 1 in

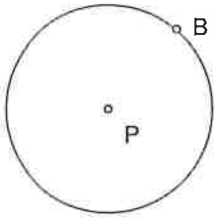
$k = 2$



Radii
pre-image = 1/2 in image = 2 in

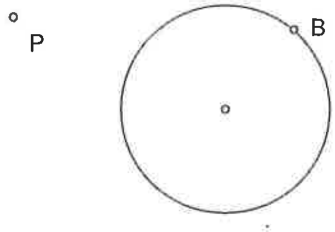
Set 2

$k = 3$



Radii
pre-image = image =

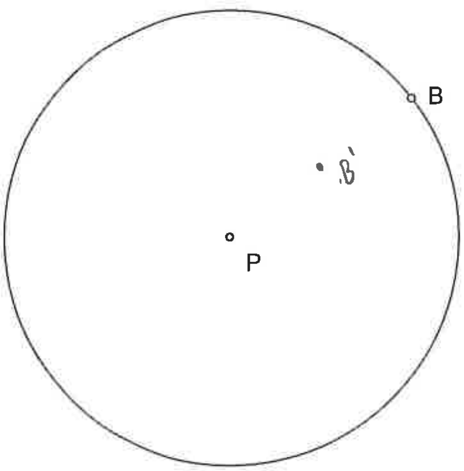
$k = 3$



Radii
pre-image = image =

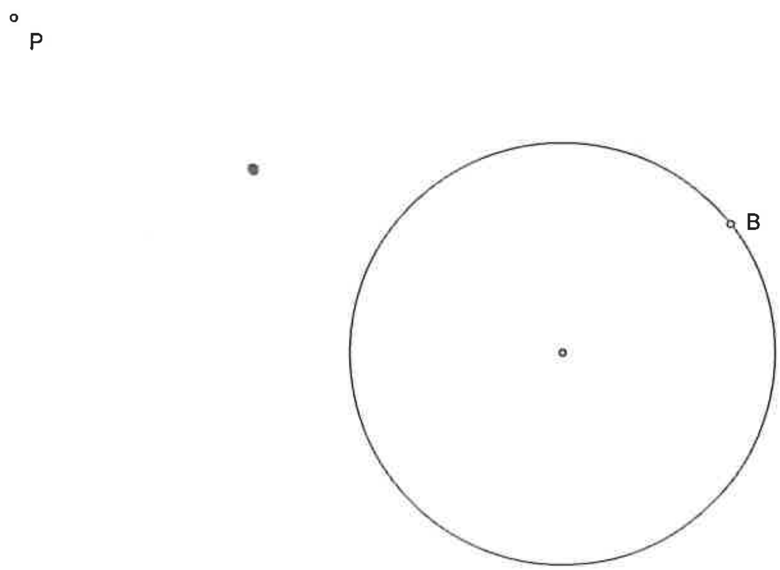
Set 3

$k = ½$



Radii
pre-image = image =

$k = ½$



Radii
pre-image = image =

Set 4 – Compare sets 1, 2, and 3. Which set shrunk the circle?

Circle all scale factors from the list below that would shrink an object when conducting a dilation.

¾	5	10.5	0.2
7	$^1/_5$	$^8/_3$	0.95

Compare the radii in each set. Which ratio would produce the scale factor for each set?

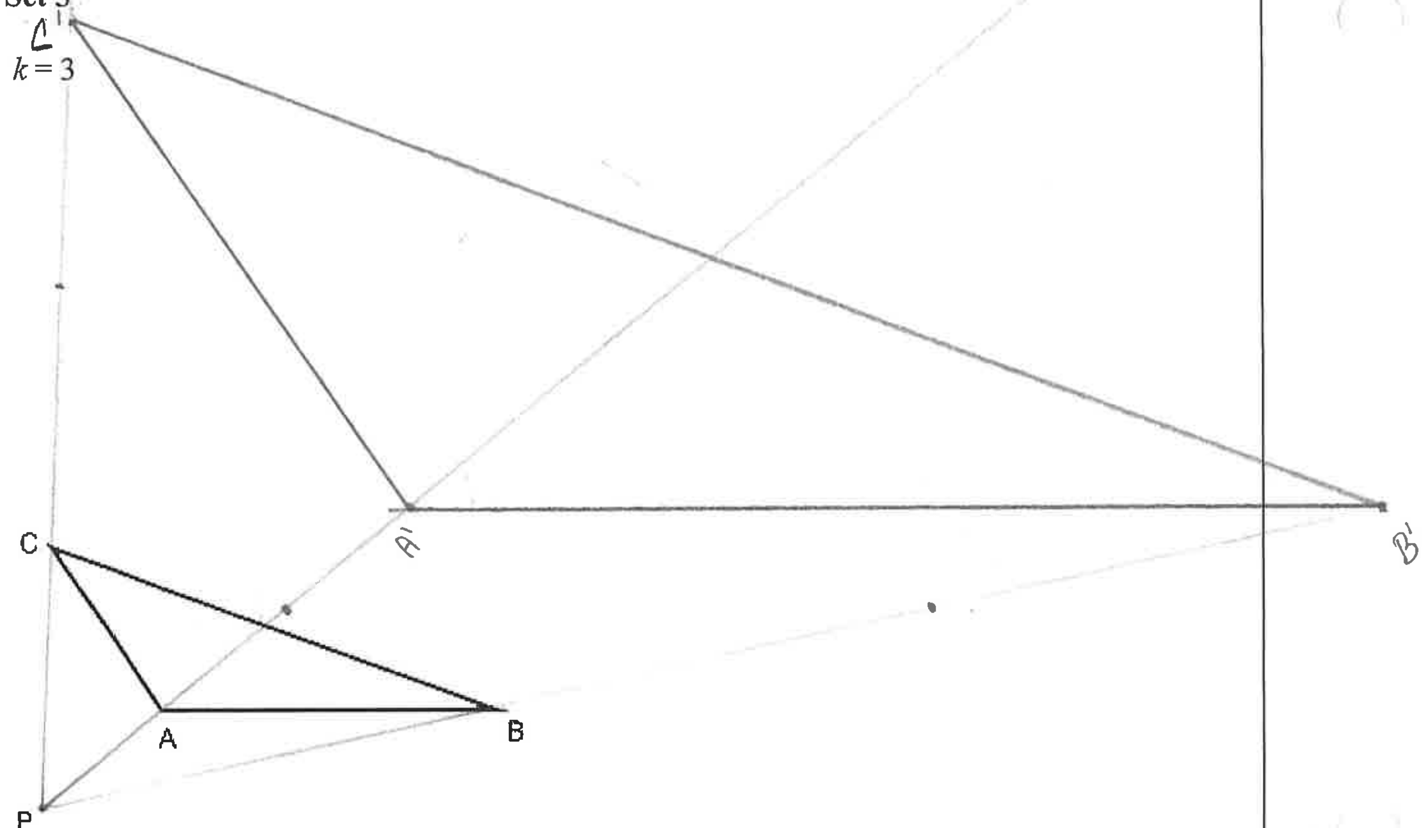
$$\frac{radius\ of\ image}{radius\ of\ pre\text{-}image} \quad \text{or} \quad \frac{radius\ of\ pre\text{-}image}{radius\ of\ image}$$

Justify your selection using the radii and scale factor from set 1.

Dilate the triangles below. Use the scale factor, k, and point P as the center of dilation. Label the resulting images appropriately. Answer all questions.

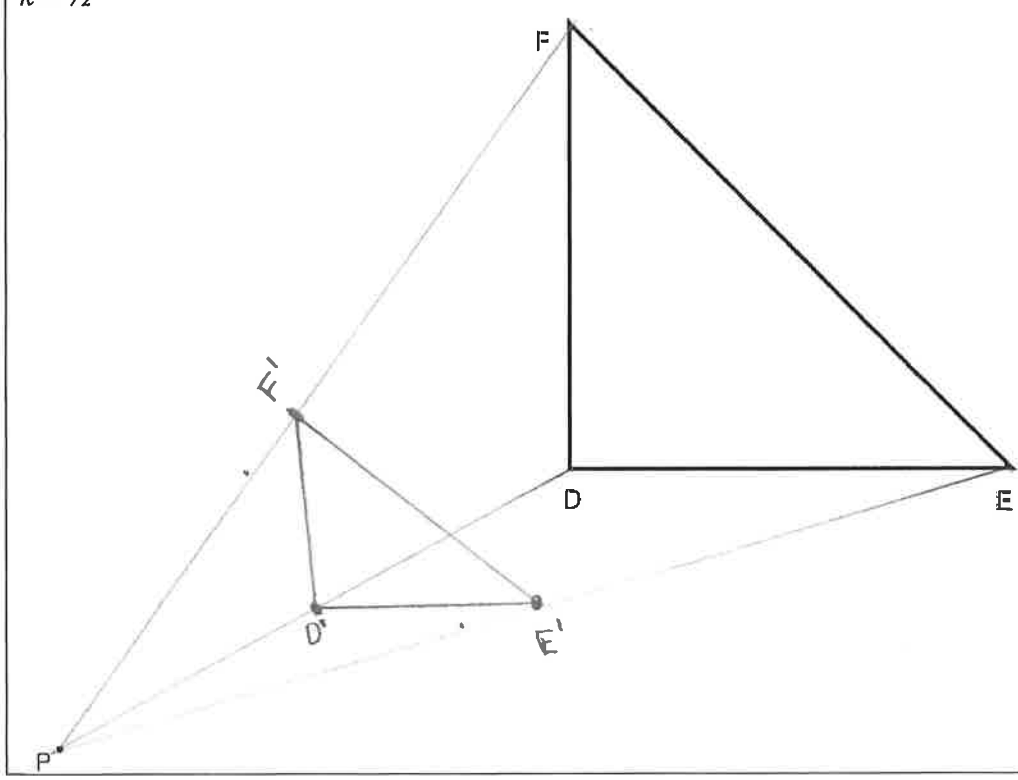
Set 5

$k = 3$

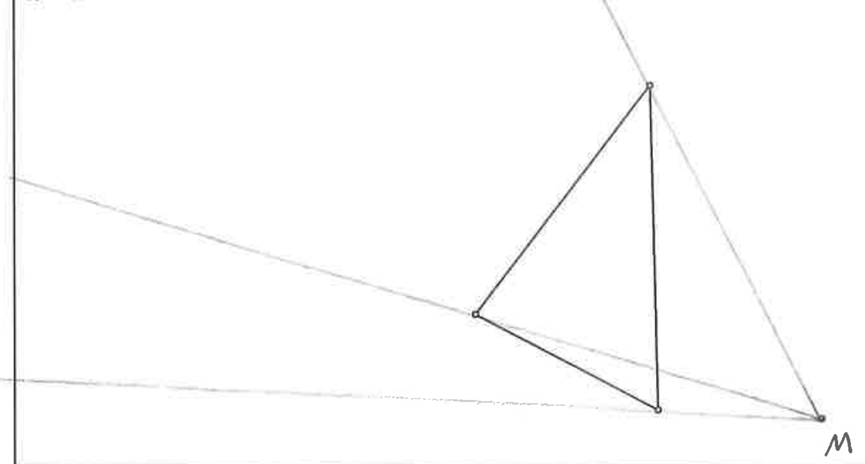
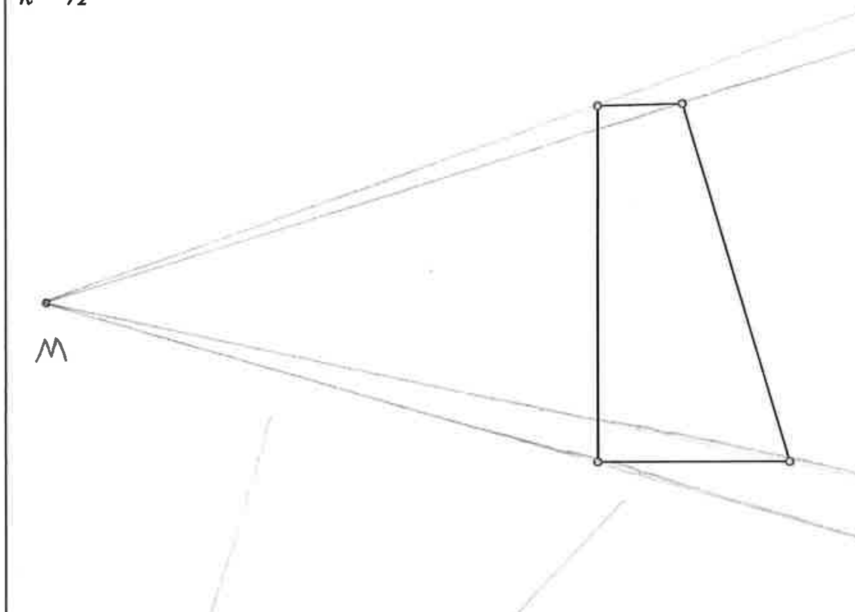
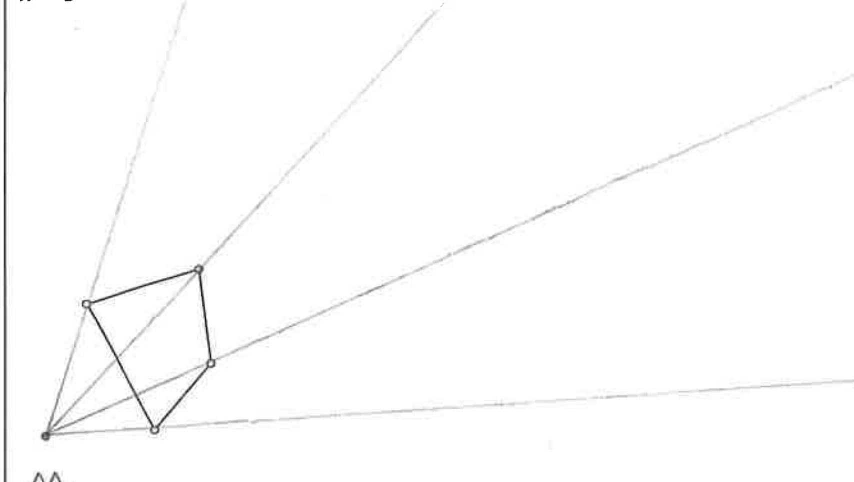


Set 6

$k = \frac{1}{2}$



Review – Perform a dilation on the object using the given scale factor k and point M as the center of dilation. Answer any questions.

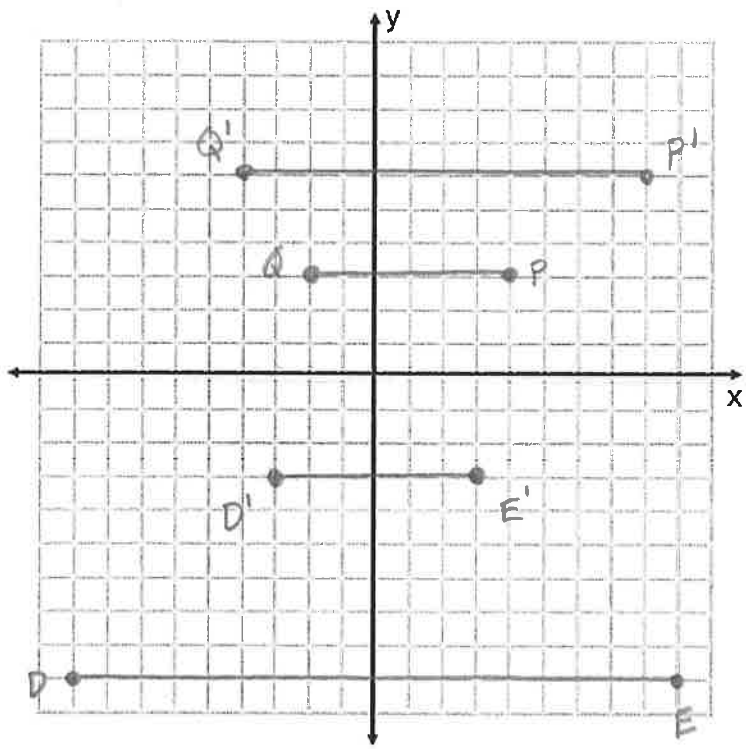
R#1 $k = 2$ 	A circle with a radius 6 inches is dilated so that its image contains a radius of 21 inches. Determine the scale factor that was used to dilate the circle. $6k = 21$ $k = \frac{21}{6}$
R#2 $k = ½$ 	A triangle with a base of 9 cm is dilated using a scale factor of 5. What will be the measure of the base of its image? 45cm
R#3 $k = 3$ 	A square has been dilated. The image of the square has a perimeter of 25. The pre-image has a perimeter of 125. Find the scale factor that was used to dilate the square. $125k = 25$ $k = \frac{25}{125}$ $k = \frac{1}{5}$

Lesson 2.12 – Dilations in the Coordinate Plane

In this section we are going to take a closer look at dilations in the coordinate plane. We will use line segments and triangles. Before we begin, complete the following:

How would you describe a line segment?	State the amount of degrees the three angle of a triangle add up to.
How many quadrants are there in the Cartesian Plane?	How do we typically express a point when using an *x* axis and *y* axis?

SET 1 - Graph

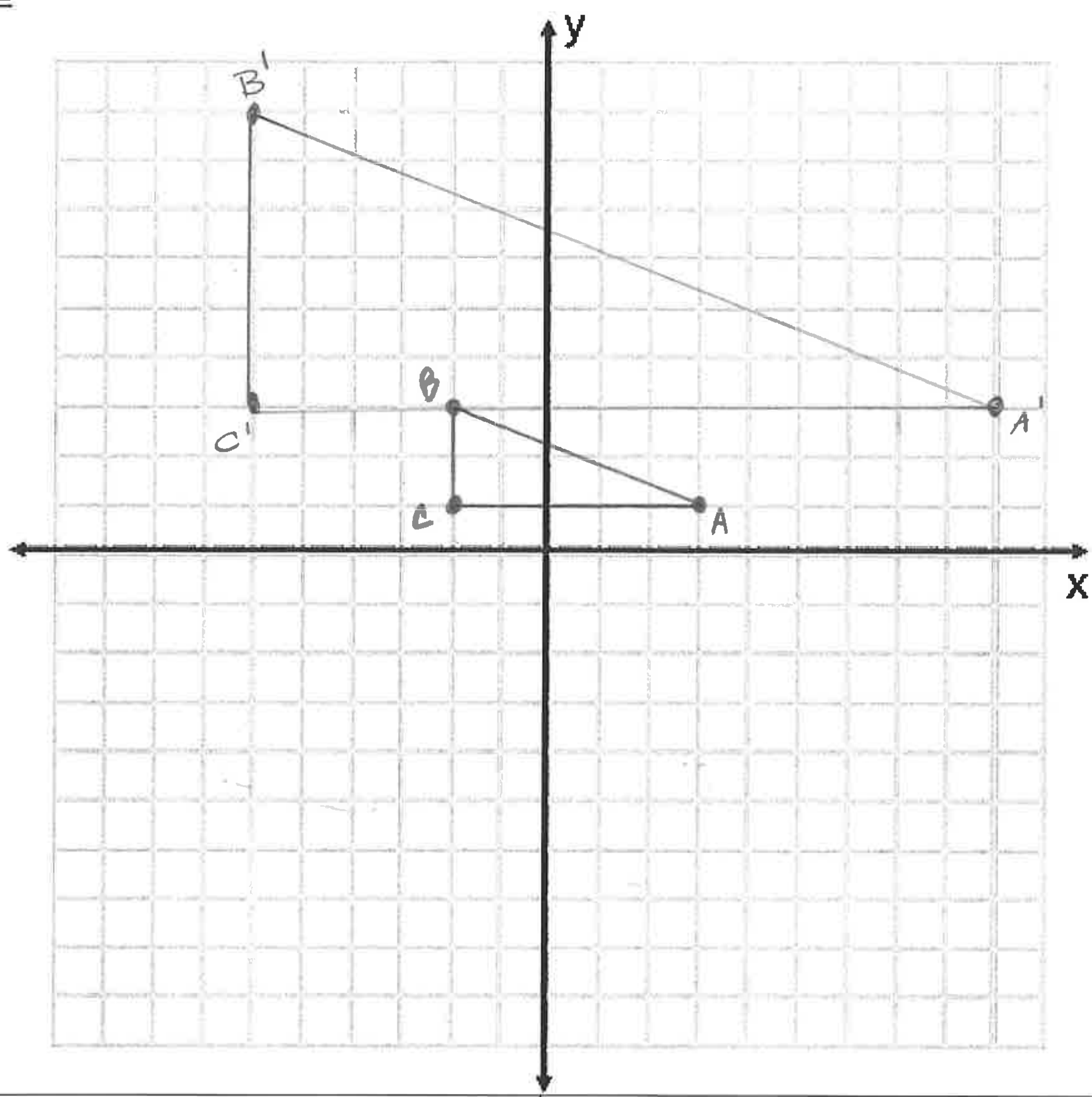


Part 1 • Plot the points P(4,3) and Q(-2,3). • Connect the points to form line segment PQ • Dilate line segment PQ using the origin as the center of dilation and a scale factor of 2. • Label the new image P'Q'. $P(4,3) \xrightarrow{\times 2} P'(8,6)$	**Part 2** • Plot the points D(-9,-9) and E(9,-9). • Connect the points to form line segment DE • Dilate line segment DE using the origin as the center of dilation and a scale factor of $\frac{1}{3}$. • Label the new image D'E'. $D(-9,-9) \xrightarrow{\times 1/3} D'(-3,-3)$

$Q(-2,3) \xrightarrow{\times 2} Q'(-4,6)$

$E(9,-9) \xrightarrow{\times 1/3} E'(3,-3)$

SET 2



- Plot the points A(3,1), B(-2,3), and C(-2,1). Connect the points to form triangle ABC.

- Dilate triangle ABC using the origin as the center of dilation and a scale factor of 3.

$A(3,1) \xrightarrow{\times 3} A'(9,3)$

$B(-2,3) \xrightarrow{\times 3} B'(-6,9)$

$C(-2,1) \xrightarrow{\times 3} C'(-6,3)$

- Show how line segment A'B' is three times as large as line segment AB.

- Measure all the angles in the original triangle and state them below. Measure all the angles in the 'prime' triangle and state them below.

  a) ∠A = _______

  b) ∠B = _______

  c) ∠C = _______

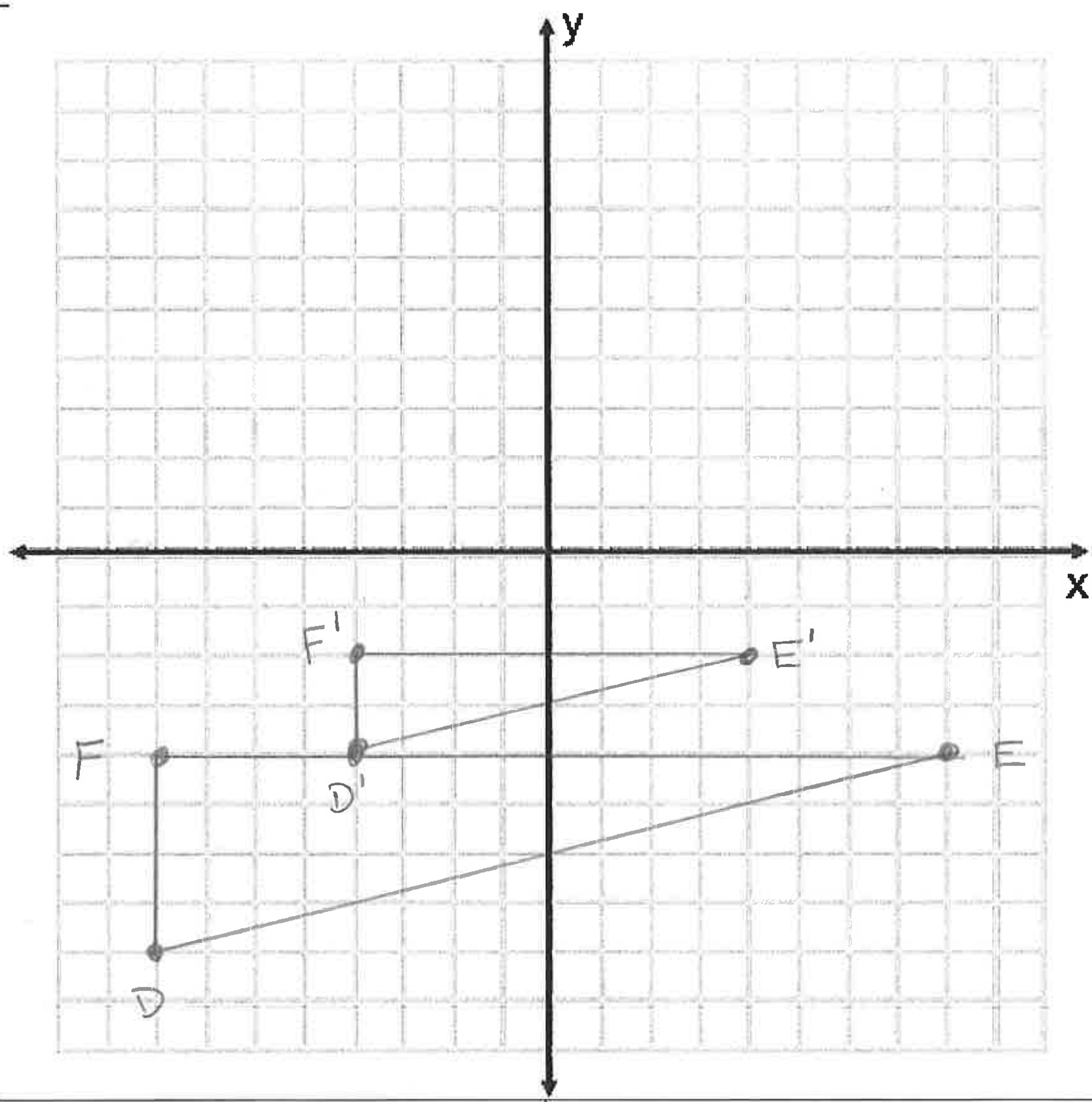
  d) ∠A' = _______

  e) ∠B' = _______

  f) ∠C' = _______

Have the angles changed measure?

SET 3



- Plot the points D(-8,-8), E(8,-4), and F(-8,-4). Connect the points to form triangle DEF.
- Dilate triangle DEF using the origin as the center of dilation and a scale factor of 0.5. State the original and the resulting coordinates below.
  - D(-8 ,-8) → D'(-4 ,-4)
  - E(8 ,-4) → E'(4 ,-2)
  - F(-8 , -4) → F'(-4 ,-2)
- Compare the length of DE and D'E'. Explain what you find.

- Measure all the angles in the original triangle and state them below. Measure all the angles in the 'prime' triangle and state them below.
  a) ∠D = _______
  b) ∠E = _______
  c) ∠F = _______
  d) ∠D' = _______
  e) ∠E' = _______
  f) ∠F' = _______

Have the angles changed measure?

Review - Follow the instructions in each review.

R#1

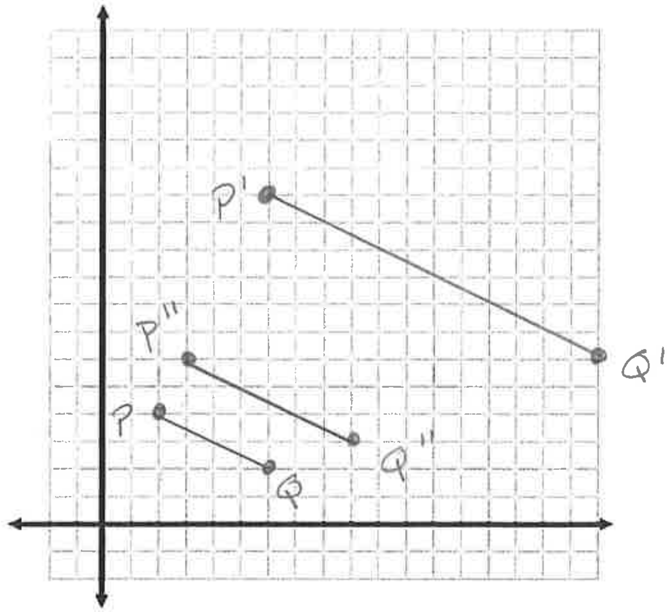
- Plot and label the points P(2,4) and Q(6,2). Connect the points to form line segment PQ.
- Dilate PQ using a scale factor of 3. State the coordinates and label the image P'Q'.
- Dilate P'Q' using a scale factor of ½ . State the coordinates and label the image P"Q".

$P(2,4) \longrightarrow P'(6,12)$

$Q(6,2) \longrightarrow Q'(18,6)$

$P'(6,12) \longrightarrow P''(3,6)$

$Q'(18,6) \longrightarrow Q''(9,3)$



R#2

- Plot the points A(-9,-3), B(-15,-18), and C(--6,-12). Connect the points to form triangle ABC.
- Dilate triangle ABC using a scale factor of 1/3. State the coordinates and label the image A'B'C'.
- Determine the scale factor needed to dilate point B' so that its image is point (-10,-6).

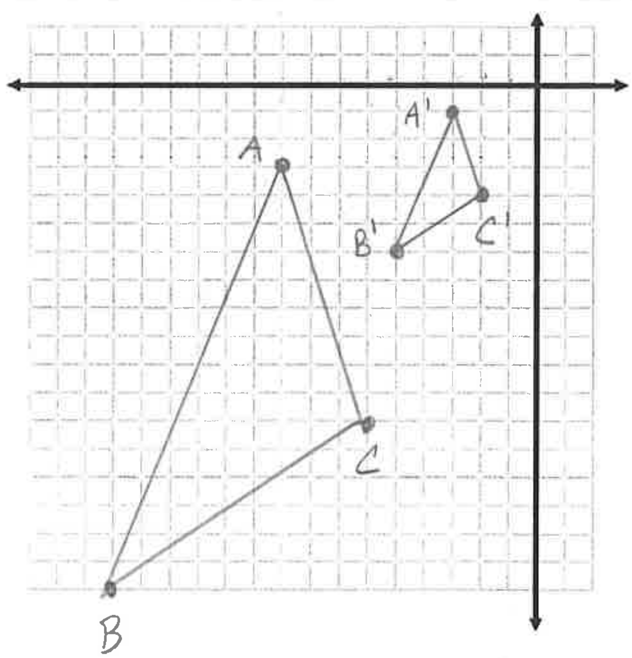
$A(-9,-3) \longrightarrow A'(-3,-1)$

$B(-15,-18) \longrightarrow B'(-5,-6)$

$C(-6,-12) \longrightarrow C'(-2,-4)$

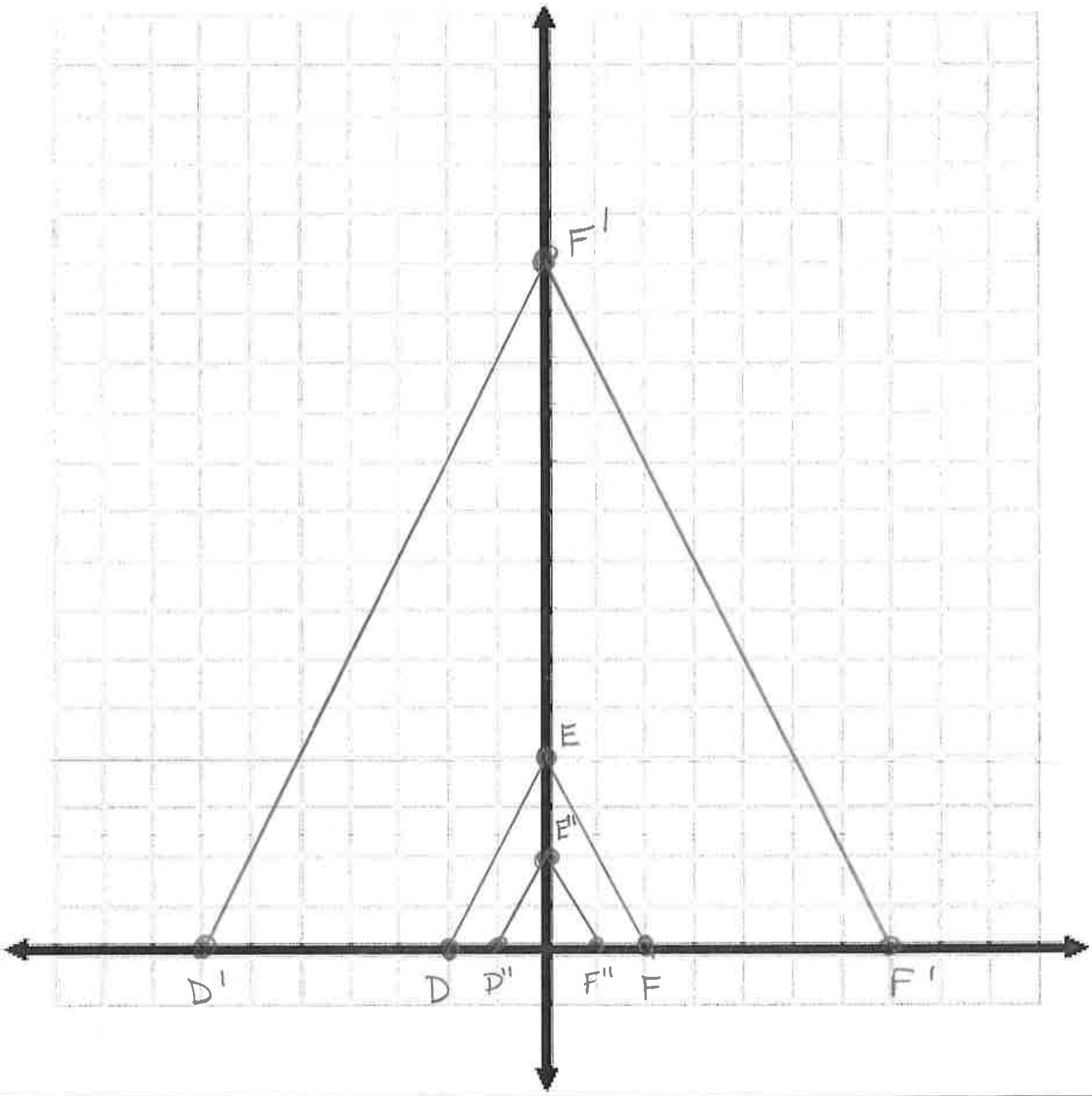
$B'(-5,-6) \xrightarrow{\times 2} B''(-10,-12)$

$K = 2$



R#3

- Plot the points D(-2,0), E(0,4), and F(2,0). Connect the points to form triangle DEF.
- Dilate triangle DEF using a scale factor of 3.5. State the coordinates and label the image D'E'F'.
- Dilate D'E'F' so that its image lies inside of triangle DEF. State the scale factor used.



$D(-2,0) \longrightarrow D'(-7,0)$
$E(0,4) \longrightarrow E'(0,14)$
$F(2,0) \longrightarrow F'(7,0)$

use scale factor of $1/7$
$D'(-7,0) \longrightarrow D''(-1,0)$
$E'(0,14) \longrightarrow E''(0,2)$
$F'(7,0) \longrightarrow F''(1,0)$